Pascal's Triangle

Binomial Expansions

Expand and simplify each of the following powers of $(x+y)$:

$(x+y)^0 = 1$ row 0

$(x+y)^1 = 1x + 1y$ row 1

$(x+y)^2 = 1x^2 + 2xy + 1y^2$ row 2

$(x+y)^3 = (x+y)(x^2+2xy+y^2)$
$= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$
$= 1x^3 + 3x^2y + 3xy^2 + 1y^3$ row 3

$(x+y)^4 = (x+y)(x^3+3x^2y+3xy^2+y^3)$
$= x^4 + 3x^3y + 3x^2y^2 + xy^3 + x^3y + 3x^2y^2 + 3xy^3 + y^4$
$= 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$ row 4

Examine the coefficients of the terms for each expansion. What do you notice?

The coefficients for each expansion correspond to a row of Pascal's Triangle.
(the exponent of the binomial matches the row number)

Examine the variables and exponents in the expansion of $(x+y)^4$. What pattern do you notice about the powers of x? What pattern do you notice about the powers of y?

The powers of x descend from exponent 4 to 0 while the powers of y ascend from exponent 0 to 4. The sum of the exponents is always 4 for each term. There are (4+1) terms in total.

Predict the expansion of $(x+y)^5$ using the above observations.

$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

The expansion of $(x+y)^n$ is of the form…

$$a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + a_3x^{n-3}y^3 + \cdots + a_{n-1}xy^{n-1} + a_ny^n$$

where the coefficients of each term correspond to… elements in row "n" of Pascal's Triangle.

Ex 1: Expand each of the following:

a) $(2x+y)^5$

$$= (2x)^5 + 5(2x)^4(y) + 10(2x)^3(y)^2 + 10(2x)^2(y)^3 + 5(2x)^1(y)^4 + (y)^5$$

$$= 2^5x^5 + 5\cdot 2^4x^4y + 10\cdot 2^3x^3y^2 + 10\cdot 2^2x^2y^3 + 5\cdot 2xy^4 + y^5$$

$$= 32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$$

b) $(x-2y)^6$

$$= (x)^6 + 6(x)^5(-2y) + 15(x)^4(-2y)^2 + 20(x)^3(-2y)^3 + 15(x)^2(-2y)^4 + 6(x)(-2y)^5 + (-2y)^6$$

$$= x^6 + 6(-2)x^5y + 15(-2)^2x^4y^2 + 20(-2)^3x^3y^3 + 15(-2)^4x^2y^4 + 6(-2)^5xy^5 + (-2)^6y^6$$

$$= x^6 - 12x^5y + 60x^4y^2 - 160x^3y^3 + 240x^2y^4 - 192xy^5 + 64y^6$$