Day 6: 3.4 – Solving Rational Equations & Inequalities

EX 1 – Solving a rational equation

- Factor the expressions
- Cross multiply & solve for the variable

a) $\dfrac{4}{3x-5}=4$, $x \neq \dfrac{5}{3}$

$4 = 4(3x-5)$

$1 = 1(3x-5)$

$3x-5=1$

$3x=6$

$\boxed{x=2}$

b) $\dfrac{x-5}{x^2-3x-4}=\dfrac{3x+2}{x^2-1}$

$\dfrac{x-5}{(x-4)(x+1)} = \dfrac{3x+2}{(x-1)(x+1)}$, $x \neq \pm 1, 4$

$(x-5)(x-1) = (3x+2)(x-4)$

$x^2-x-5x+5 = 3x^2-12x+2x-8$

$2x^2-4x-13=0$

does not factor. QF: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{4 \pm \sqrt{4-4(2)(-13)}}{4}$

$= \dfrac{4 \pm \sqrt{4+104}}{4}$

$= \dfrac{4 \pm \sqrt{108}}{4} = \dfrac{4 \pm 2\sqrt{27}}{4}$

$= \dfrac{2 \pm \sqrt{27}}{2}$

EX 2- Solving a rational inequality using an interval table

- Gather all terms to one side of inequality
- Factor and simplify
- Create an interval table using x-intercepts and vertical asymptotes
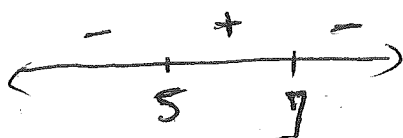
a) $\dfrac{2}{x-5} < 1$

$\dfrac{2}{x-5} - 1 < 0$

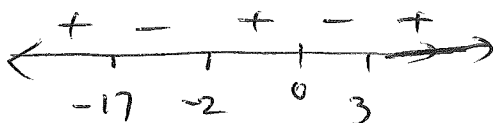$\dfrac{2}{x-5} - \dfrac{1(x-5)}{x-5} < 0$

$\dfrac{2-x+5}{(x-5)} < 0$

$\dfrac{-(x-7)}{x-5} < 0$
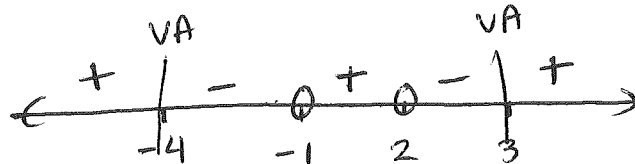

$$x \in (-\infty, 5) \cup (7, \infty)$$

c) $\dfrac{2x}{x-3} \leq \dfrac{3x}{2x+4}$

$$x \in [-17, -2) \cup [0, 3)$$

b) $\dfrac{x^2 - x - 2}{x^2 + x - 12} \geq 0$

$\dfrac{(x-2)(x+1)}{(x+4)(x-3)} \geq 0$

$$x \in (-\infty, -4) \cup [1, 2] \cup (3, \infty)$$

NOTE: $x = -4, 3$ cannot be part of the solution, since they are restrictions.
(VA on the graph)

$\dfrac{2x}{x-3} - \dfrac{3x}{2x+4} \leq 0$

$\dfrac{2x(2x+4) - 3x(x-3)}{(x-3)(2x+4)} \leq 0$

$\dfrac{4x^2 + 8x - 3x^2 + 9x}{(x-3)(2x+4)} \leq 0$

$\dfrac{x^2 + 17x}{(x-3)(2x+4)} \leq 0$