Day 5 – Compound Angle Formulas

A **compound angle formula** is a trigonometric expression that depends on two or more angles. By using compound angle formulae, we can determine exact values of trig rations that **are not multiples of special angles.** They will be expressed as the sum or difference of the special angles.

The compound angle, or addition and subtraction formulas are:

$\sin(x+y) = \sin x\cos y + \cos x\sin y$
$\sin(x-y) = \sin x\cos y - \cos x\sin y$
$\cos(x+y) = \cos x\cos y - \sin x\sin y$
$\cos(x-y) = \cos x\cos y + \sin x\sin y$

$$tan(x+y) = \frac{tanx+tany}{1-(tanx)(tany)}$$

$$tan(x-y) = \frac{tanx-tany}{1+(tanx)(tany)}$$

EX 1 – Show that the subtraction formula for sine is true for $x = \frac{\pi}{2}$ and $y = \frac{\pi}{4}$

$$\sin(x-y) = \sin x\cos y - \cos x\sin y$$

$$\sin\left(\frac{\pi}{2}-\frac{\pi}{4}\right) = \sin\frac{\pi}{2}\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\sin\frac{\pi}{4}$$

$$= (1)\left(\frac{\sqrt{2}}{2}\right) - (0)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{\pi}{2}-\frac{\pi}{4}\right)$$

$$= \sin\left(\frac{2\pi}{4}-\frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

EX 2 - Determine an exact value for: $\cos\left(\frac{3\pi}{4}+\frac{\pi}{6}\right)$

$$= \cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$$

$$= -\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{-(\sqrt{6}+\sqrt{2})}{4}$$

EX 3 - Determine the exact value for each of the following:

a) $\cos 75^\circ = \cos(45^\circ + 30^\circ)$

$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

b) $\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

(WE KNOW $\cos 75^\circ = \sin 15^\circ$!!)

EX 4 - Use an appropriate compound angle formula to determine the exact value for: $\sin\frac{23\pi}{12}$

$\frac{23\pi}{12} = 2\pi - \frac{\pi}{12}$ $\quad \therefore \sin\frac{23\pi}{12} = -\sin\frac{\pi}{12}$ $\quad \left[\frac{23\pi}{12} \text{ in } Q4\right]$

$\therefore \sin\frac{23\pi}{12} = -\sin\frac{\pi}{12} = -\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

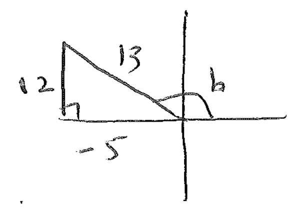
$= -\left[\sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}\right]$

$= -\left[\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}\right]$

$= -\left[\frac{\sqrt{6}-\sqrt{2}}{4}\right] = \frac{\sqrt{2}-\sqrt{6}}{4}$

EX 5 - **If** $\sin a = -\frac{4}{5}$, $\pi \le a \le \frac{3\pi}{2}$ **and** $\cos b = -\frac{5}{13}$, $\frac{\pi}{2} \le b \le \pi$, **evaluate** $\tan(a+b)$.

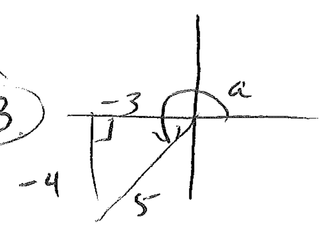
Q2



$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

$= \frac{\frac{4}{3} + \frac{12}{-5}}{1 - \left(\frac{4}{3}\right)\left(-\frac{12}{5}\right)}$

Q3



$= \frac{20-36}{15} \div \left[\frac{15+48}{15}\right]$

$= \frac{-16}{15} \times \frac{15}{63} = \frac{-16}{63}$

EXTRA PRACTICE – COMPOUND ANGLE FORMULAS

1. Determine the value, using compound addition identities, of each of the following:

a) $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

b) $\cos 165^\circ = \cos(180-15)$

$= -\cos 15$

$= -\left[\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)\right]$

$= -\left[\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}\right]$

$= -\left[\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}\right]$

$= -\left[\frac{\sqrt{6}+\sqrt{2}}{4}\right]$

2. Express as a single trigonometric function.

a) cos2A cosA - sin2A sinA

$= \cos(2A+A)$

$= \cos(3A)$

b) sin5 cos2 - sin2 cos5

$= \sin(5-2)$

$= \sin 3$

c) $\frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$

$= \tan(2x+3x)$

$= \tan(5x)$

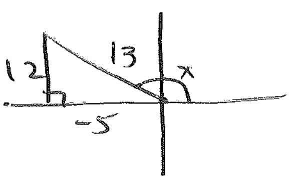
d) $\cos^2(x+y) + \sin^2(x+y)$

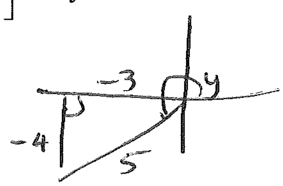
$= \cos((x+y) - (x+y))$

$= \cos(0)$

$= 1$

3. If x is in the interval $\left[\frac{\pi}{2}, \pi\right]$ and y is in the interval $\left[\pi, \frac{3\pi}{2}\right]$, and $\cos x = \frac{-5}{13}$, and $\tan y = \frac{4}{3}$, evaluate the following:





a) $\sin(x-y)$

$= \sin x \cos y - \cos x \sin y$

$= \left(\frac{12}{13}\right)\left(\frac{-3}{5}\right) - \left(\frac{-5}{13}\right)\left(\frac{-4}{5}\right)$

$= \frac{-36}{65} - \frac{20}{65}$

$= \frac{-56}{65}$

b) $\cos(x+y)$

$= \cos x \cos y - \sin x \sin y$

$= \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right) - \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right)$

$= \frac{15}{65} + \frac{48}{65}$

$= \frac{63}{65}$

c) $\tan(x-y)$

$= \frac{\tan x - \tan y}{1 + (\tan x)(\tan y)}$

$= \frac{\left(\frac{-12}{5}\right) - \left(\frac{4}{3}\right)}{1 + \left(\frac{-12}{5}\right)\left(\frac{4}{3}\right)}$

$= \frac{-56}{15} \div \left[\frac{15-48}{15}\right]$

$= \frac{-56}{-33} = \frac{56}{33}$