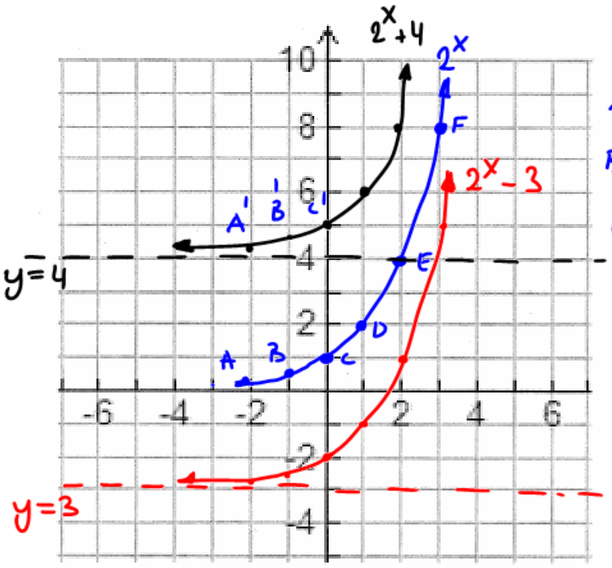
Transforming Exponential Functions

1. Using mapping notation , sketch the graph of $f(x) = 2^x$, $g(x) = 2^x + 4$ and $h(x) = 2^x - 3$.

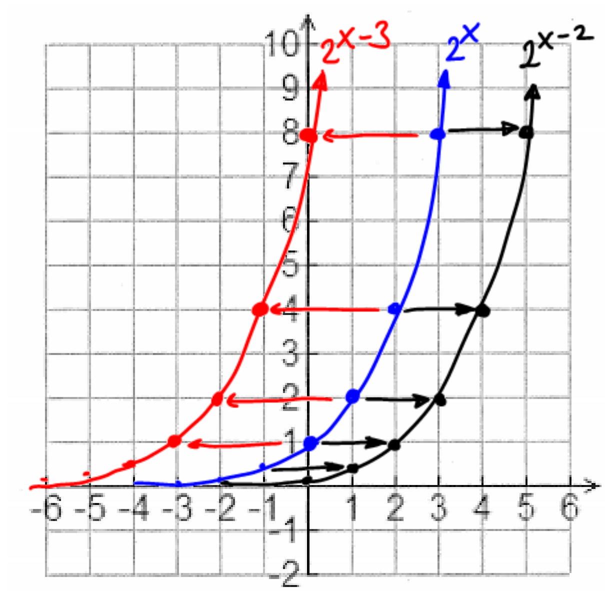


$(x,y) \rightarrow (x, y+4)$
$A(-2, 0.25) \rightarrow A'(-2, 4.25)$
$B(-1, 0.5) \rightarrow B'(-1, 4.5)$
$C(0, 1) \rightarrow C'(0, 5)$
$D(1, 2) \rightarrow D'(1, 6)$
$E(2, 4) \rightarrow E'(2, 8)$

$(x,y) \rightarrow (x, y-3)$
move all the points 3 units down

	Domain	Range	y-intercept	Asymptote
$f(x) = 2^x$	$\{x \in R\}$	$\{y \in R \mid y > 0\}$	$(0, 1)$	$y = 1$
$g(x) = 2^x + 4$	$\{x \in R\}$	$\{y \in R \mid y > 4\}$	$(0, 5)$	$y = 4$
$h(x) = 2^x - 3$	"	$\{y \in R \mid y > -3\}$	$(0, -2)$	$y = -3$
$\boldsymbol{f(x) = b^x + c}$	"	$\{y \in R \mid y > c\}$	$(0, c+1)$	$y = c$

2. Using ~~MAPPING NOTATION~~, sketch the graph of $f(x) = 2^x$, $g(x) = 2^{x-2}$ and $h(x) = 2^{x+3}$.

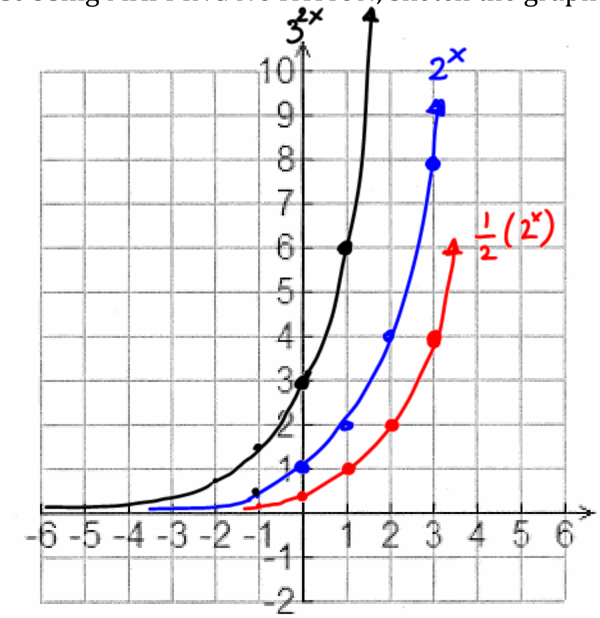


$g(x) = 2^{x-2}$
horizontal shift 2 units RIGHT

$h(x) = 2^{x+3}$
horizontal shift 3 units LEFT

	Domain	Range	y-intercept	Asymptote
$f(x) = 2^x$	$\{x \in R\}$	$\{y \in R \mid y > 0\}$	$(0, 1)$	$y = 0$
$g(x) = 2^{x-2}$	"	"	$(0, 0.25)$	$y = 0$
$h(x) = 2^{x+3}$.	"	"	$(0, 8)$	$y = 0$
$\boldsymbol{f(x) = b^{x+d}}$	"	"	$(0, b^d)$	$y = c$

3. Using ~~MAPPING NOTATION~~, sketch the graph of $f(x) = 2^x$, $g(x) = 3(2^x)$, and $h(x) = \frac{1}{2}(2^x)$.



$g(x) = 3(2^x)$

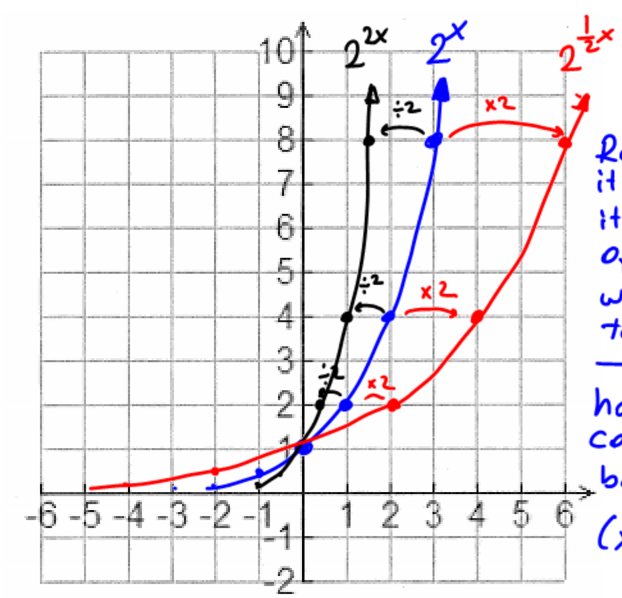
Vertical stretch bafo 3 (y values triple)

$h(x) = \frac{1}{2}(2^x)$

vertical compression bafo 0.5 (y values halved)

	Domain	Range	y-intercept	Asymptote
$f(x) = 2^x$	$\{x \in R\}$	$\{y \in R \mid y > 0\}$	$(0, 1)$	$y = 0$
$g(x) = 3(2^x)$	"	"	$(0, 3)$	"
$h(x) = \frac{1}{2}(2^x)$	"	"	$(0, 0.5)$	"
$\boldsymbol{f(x) = a(b^x)}$	"	"	$(0, a)$	$y = c$

4. Using MAPPING NOTATION, sketch the graph of $f(x) = 2^x$, $g(x) = 2^{2x}$, and $h(x) = 2^{\frac{1}{2}x}$.



$g(x) = 2^{(2x)}$

Remember when it's horizontal, it's always the opposite of what it seems to appear

horizontal compression bafo ½ (x value halved)

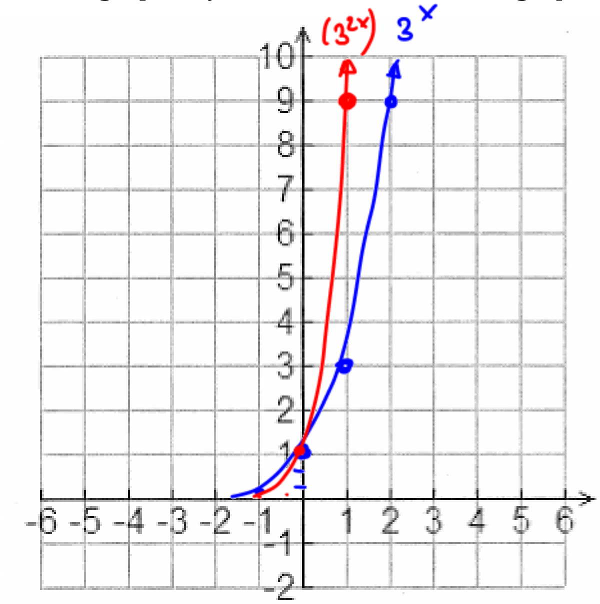
$h(x) = 2^{(\frac{1}{2}x)}$

RED

horizontal stretch bafo 2 (x values double)

	Domain	Range	y-intercept	Asymptote
$f(x) = 2^x$	$\{x \in R\}$	$\{y \in R \mid y > 0\}$	$(0, 1)$	$y = 0$
$g(x) = 2^{2x}$	"	"	"	"
$h(x) = 2^{\frac{x}{2}}$	"	"	"	"
$\boldsymbol{f(x) = b^{kx}}$	"	"	$(0, c+1)$	$y = c$

Use the graph of $f(x) = 3^x$ to sketch the graph of $g(x) = 3^{2x}$.



$g(x) = 3^{2x}$

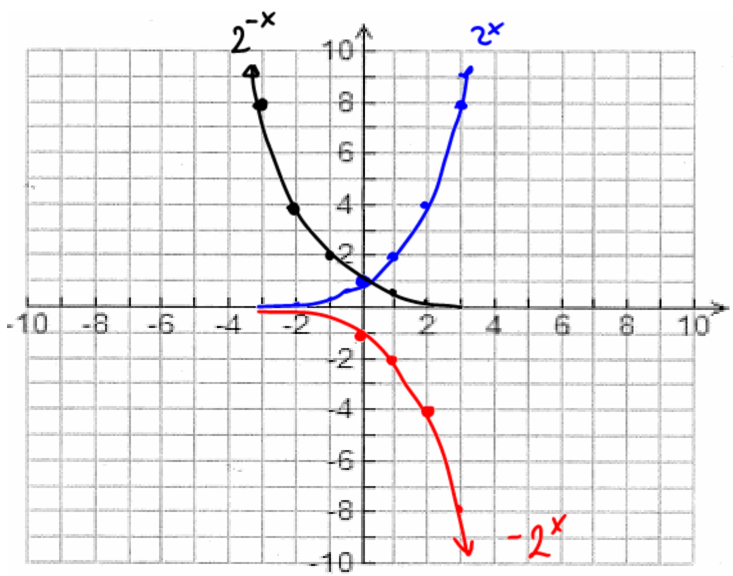
horizontal compression by a factor of ½ (x values halved)

OR

$g(x) = (3^2)^x$
$= (9)^x$

graph this

5. **a.** Use the graph of $f(x) = 2^x$ to sketch the graph of $g(x) = 2^{-x}$ and $h(x) = -2^x$



$g(x) = 2^{-x}$ → OR → $g(x) = (2^{-1})^x$
$= (\frac{1}{2})^x$

horizontal reflection over "y" axis

$h(x) = -2^x$

vertical reflection over "x" axis

	Domain	Range	y-intercept	Asymptote
$f(x) = 2^x$	$\{x \in R\}$	$\{y \in R \mid y > 0\}$	$(0,1)$	$y = 0$
$g(x) = 2^{-x}$	"	"	"	"
$h(x) = -2^x$	"	$\{y \in R \mid y < 0\}$	$(0,-1)$	$y = 0$
$\boldsymbol{f(x) = b^{-x}}$	"	$\{y \in R \mid y > 0\}$	$(0,1)$	
$\boldsymbol{f(x) = -b^x}$	"	$\{y \in R \mid y < 0\}$	$(0,-(+1))$	$y = 0$

"−" direction

6. State the MAPPING NOTATION, and then **describe** the transformations:

a. $f(x) = 3(2^{-x+2}) - 1$ factor

$= 3\left[2^{-(x-2)}\right] - 1$

$(x,y) \rightarrow (kx+d, ay+c)$

$(x,y) \rightarrow (-x+2, 3y-1)$

R = horizontal reflection over "y" axis

S = vertical stretch by a factor of 2

T = 2 units right

1 unit down

b. $f(x) = -4^{-2x+2} + 7$

$= -4^{-2(x-1)} + 7$

$(x,y) \rightarrow (\frac{x}{-2} + 1, -y+7)$

$a = -1$ $k = -1/2$

$c = 7$ $d = 1$

R = vertical reflection over "x" axis

horizontal " " "y" "

S = horizontal compression by a factor of 1/2

T = 1 unit right

7 units up.

General form of transformed exponential function:

$$y = a\left[b^{k(x-d)}\right] + c$$

Effect of :

y coordinate

a:* when i) $a > 1$, *it is a vertical stretch by a factor of* $|a|$** ***ex: y = 2[3^x]

ii) $0 < a < 1$, *it is a vertical compression by a factor of* $|a|$ ***ex:*** y = 0.5[3^x]

iii) $a < 0$, *it is a vertical reflection ex:* y = -2[3^x]

c:* when $c > 0$, *vertical shift "c" units up ***ex:*** y = 2[3^x] + 1

$c < 0$, *vertical shift "c" units down ex:* y = 2[3^x] - 1

x coordinate

k:* when $k > 1$, *it is a horizontal compression by a factor of* $|\frac{1}{k}|$** ***ex: $y = 3^{2x}$

$0 < k < 1$, *it is a horizontal stretch by a factor of* $|\frac{1}{k}|$ *ex:* $y = 3^{1/2x}$

d:* when $d > 0$, *horizontal shift "d" units right ***ex:*** $y = 3^{2(x-2)}$

$d < 0$, *horizontal shift "d" units left* ***ex:*** $y = 3^{2(x+2)}$

REMEMBER TO FACTOR

b:* when $b > 0$, *it is an exponential GROWTH ex: y = 2[3^x]

$b < 0$, *it is an exponential DECAY ex:* y = (1/3)^x