Translations of Sinusoidal Functions

$$f(x) = a\sin[k(x-d)] + c \text{ and } f(x) = a\cos[k(x-d)] + c$$

Part A: Horizontal Translations/ Shifts

Recall: x says something yet does the exact opposite**.**

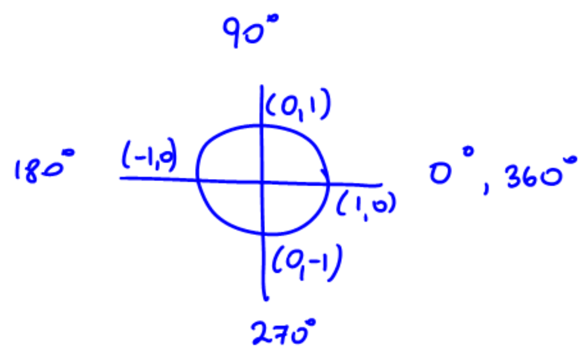
The graph of $y = f(x + d)$ is obtained from the graph of $y = f(x)$ translated d units LEFT.
The graph of $y = f(x - d)$ is obtained from the graph of $y = f(x)$ translated d units RIGHT.

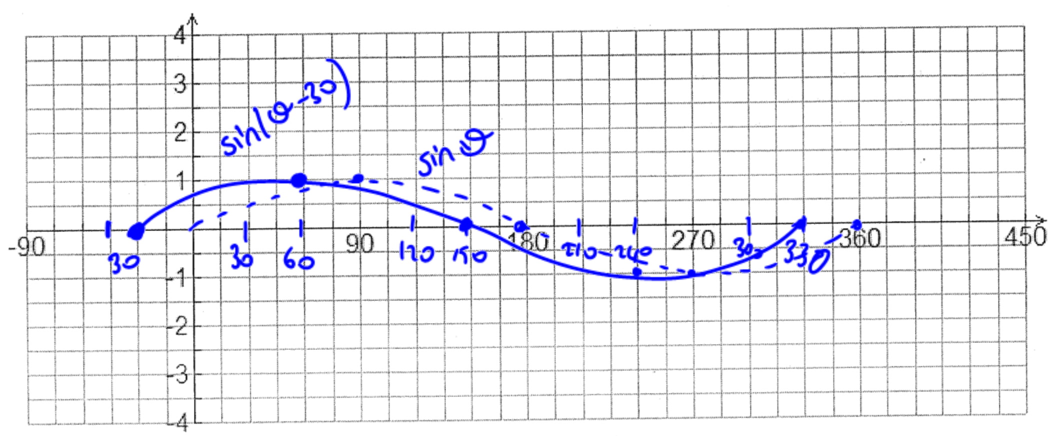
Example: Refer to $y = \sin(\theta + 30°)$ for the questions that follow.

1. Use mapping notation to graph the function below.

$\sin\theta$ (x, y) → $\sin(\theta+30)$ $(x-30, y)$

$(0,0) \rightarrow (0-30, 0) = (-30, 0)$
$(90,1) \rightarrow (90-30, 1) = (60, 1)$
$(180,0) \rightarrow (180-30, 0) = (150, 0)$
$(270,-1) \rightarrow (270-30, -1) = (240, -1)$
$(360,0) \rightarrow (360-30, 0) = (330, 0)$





Per

2. State its period and amplitude.

P = 360
a = 1

y = sin^k(θ - 30)

3. State the domain and range of the transformed function.

$D = \{\theta \in \mathbb{R}\}$ $R: \{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$

or

$D = \{\theta \in \mathbb{R} \mid 30 \leq \theta \leq 330\}$

Part B: Vertical Translations/ Shifts

The graph of $y = f(x) + c$ is obtained from the graph of $y = f(x)$ translated c units UP.
The graph of $y = f(x) - c$ is obtained from the graph of $y = f(x)$ translated c units DOWN

Example: Refer to $y = \cos\theta + 3$ for the equations that follow.

1. Use mapping notation to graph the function below.

$\cos\theta \longrightarrow \cos\theta + 3$

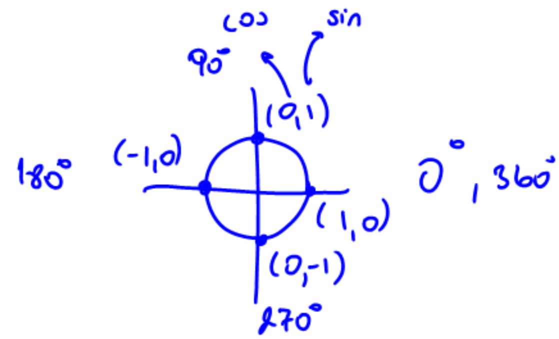
$(0, 1) \longrightarrow (0, 1+3) = (0, 4)$

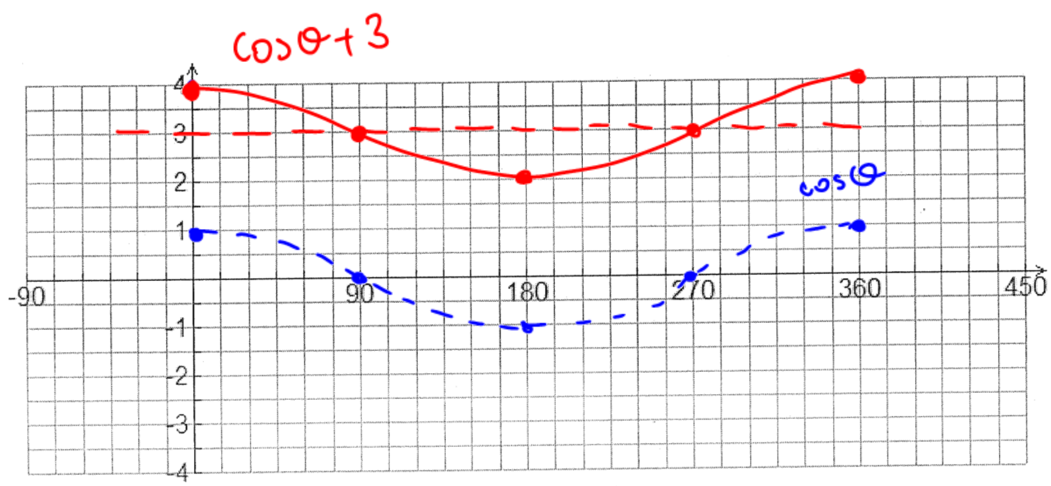
$(90, 0) \longrightarrow (90, 0+3) = (90, 3)$

$(180, -1) \longrightarrow (180, -1+3) = (180, 2)$

$(270, 0) \longrightarrow (270, 0+3) = (180, 3)$

$(360, 1) \longrightarrow (360, 1+3) = (360, 4)$





2. State its period and amplitude.

$P = 360$

$a = 1$

3. State the domain and range of the transformed function.

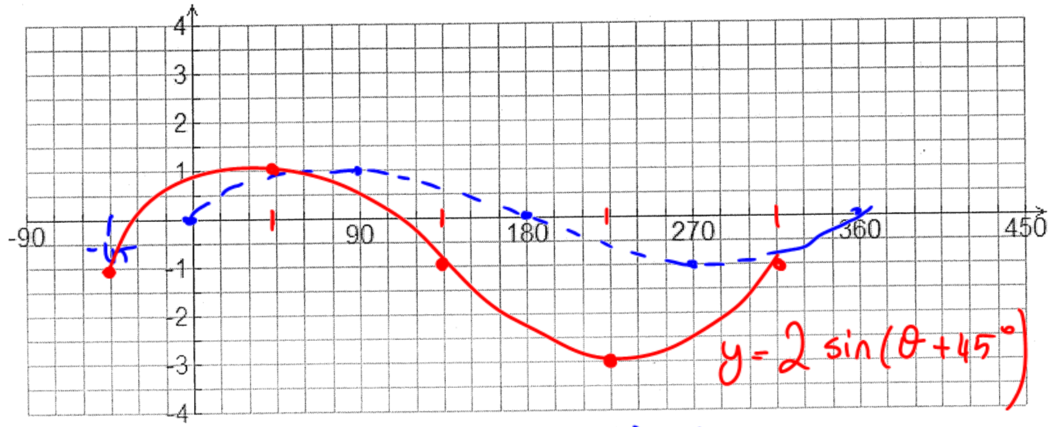
$D = \{\theta \in R\}$ $R = \{y \in R \mid 2 \leq y \leq 4\}$

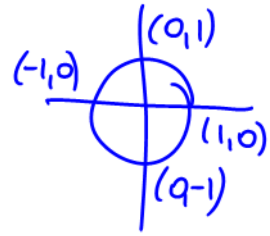
or

$D = \{\theta \in R \mid 0 \leq \theta \leq 360\}$

Let's Put it All Together!

Ex1: Graph $y = 2\sin(\theta + 45°) - \overset{1}{\cancel{2}}$ using mapping notation. Then, state its amplitude, period and equation of the axis of the curve.





sin θ $\quad$ 2 sin(θ + 45) − 1

(x, y) $\quad$ (x − 45, 2y − 1)

(0, 0) → (0 − 45, 2(0) − 1) = (−45, −1)

(90, 1) → (90 − 45, 2(1) − 1) = (45, 1)

(180, 0) → (180 − 45, 2(0) − 1) = (135, −1)

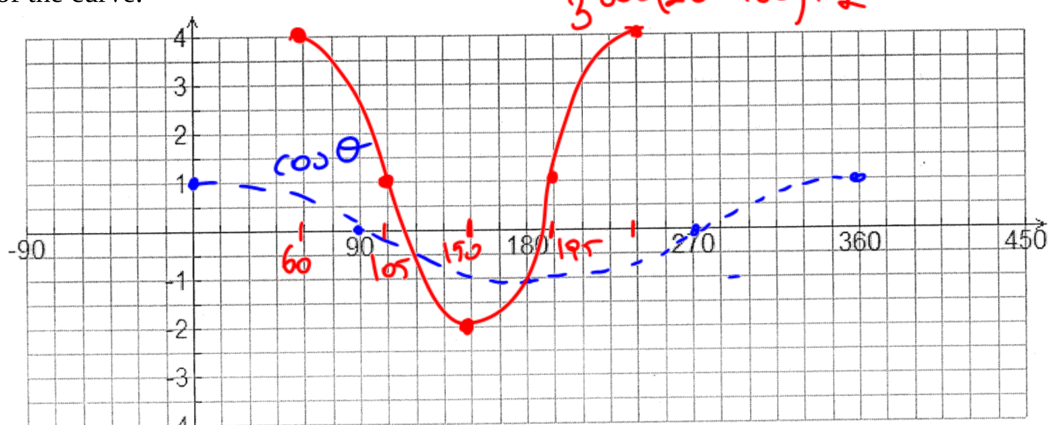
(270, −1 → (270 − 45, 2(−1) − 1) = (225, −3)

(360, 0) → (360 − 45, 2(0) − 1) = (315, −1)

Ex2: Graph $y = 3\cos(\overset{2}{}\theta - \overset{120}{\cancel{60}}°) + 1$ using mapping notation. Then, state its amplitude, period and equation of the axis of the curve.

3 cos(2θ − 120) + 1

y = 3 cos[2(θ − 60)] + 1



cos θ (x, y) $\quad$ 3 cos[2(θ − 60)] + 1 $\quad$ $\left(\frac{\theta}{2} + 60, 3y + 1\right)$

(0, 1) → $\left(\frac{0}{2} + 60, 3(1) + 1\right)$ = (60, 4)

(90, 0) → $\left(\frac{90}{2} + 60, 3(0) + 1\right)$ = (105, 1)

(180, −1) → $\left(\frac{180}{2} + 60, 3(-1) + 1\right)$ = (150, −2)

(270, 0) → $\left(\frac{270}{2} + 60, 3(0) + 1\right)$ = (195, 1)

(360, 1) → $\left(\frac{360}{2} + 60, 3(1) + 1\right)$ = (240, 4)