

GEOMETRIC SERIES

RECALL: The word "SERIES" refers to the SUM of all the terms of any kind of sequence.

- Specifically, an ARITHMETIC SERIES will be the sum of the terms that form an arithmetic sequence.
- Similarly, a GEOMETRIC SERIES will be the sum of the terms that form a geometric sequence.

THE GENERAL FORMULA

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Where a represents the 1st term of the series

n represents the number of terms in series

r represents the common ratio

EXAMPLE 1 - For the geometric series with $a = 7$ and $r = 2$, determine the ^{sum} ~~series~~ of the first 9 terms.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_9 = \frac{7(2^9 - 1)}{2 - 1}$$

$$S_9 = 3577$$

The sum of first 9 terms
is 3577

EXAMPLE 2 - Determine the sum of the geometric series $15 - 45 + 135 - 405 + \dots - 32805$

$$a = 15$$

$$r = -3$$

$$\text{let } t_n = -32805$$

$$ar^{n-1} = -32805$$

$$15(-3)^{n-1} = -32805$$

$$(-3)^{n-1} = -2187$$

$$(-3)^{n-1} = (-3)^7$$

$$n-1 = 7$$

$$n = 8$$

$$S_8 = \frac{15[(-3)^8 - 1]}{-3 - 1}$$

$$S_8 = -24600$$

Sum of series is -24600

GEOMETRIC SERIES *continued...*

EXAMPLE 3 - The first term of a sequence is 9. The terms of the sequence decrease by a factor of $\frac{1}{3}$. Find S_6 .

$$9, 6, 4, \dots$$

\therefore Geometric $a=9$
 $r=\frac{2}{3}$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \therefore S_6 = \frac{9\left[\left(\frac{2}{3}\right)^6 - 1\right]}{\frac{2}{3} - 1}$$

$$S_6 = \frac{665}{27}$$

$$\frac{1}{3}(9) = 3$$

$$\therefore 9 - 3 = 6$$

EXAMPLE 4 - The first prize in a lottery is \$100 000. Each succeeding winning number pays 40% as much as the winning number before it. How much is paid out in prizes if 6 numbers are drawn?

$$100\,000, 40\,000, 16\,000, \dots$$

$$\left. \begin{array}{l} a = 100\,000 \\ r = 0.4 \\ n = 6 \end{array} \right\} \text{geometric} \quad S_6 = \frac{100\,000(0.4^6 - 1)}{0.4 - 1}$$

$$S_6 = 165\,984$$

\therefore , \$165,984 was paid out in prize money

EXAMPLE 5 - Some companies use a telephone chain to notify employees that the company is closing because of bad weather. Suppose that, in the first round of calls, the first person in the chain calls four people. Each person called then makes four calls, and so on. What is the total number of people called in the first eight rounds of calls?

$$4, 16, 64, \dots$$

$$\begin{array}{l} a = 4 \\ r = 4 \\ n = 8 \end{array}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_8 = \frac{4(4^8 - 1)}{4 - 1}$$

$$S_8 = 87\,380$$

Yes indeed, 87 380 people were called in the first 8 rounds