Day 5: 3.3 – Special Cases

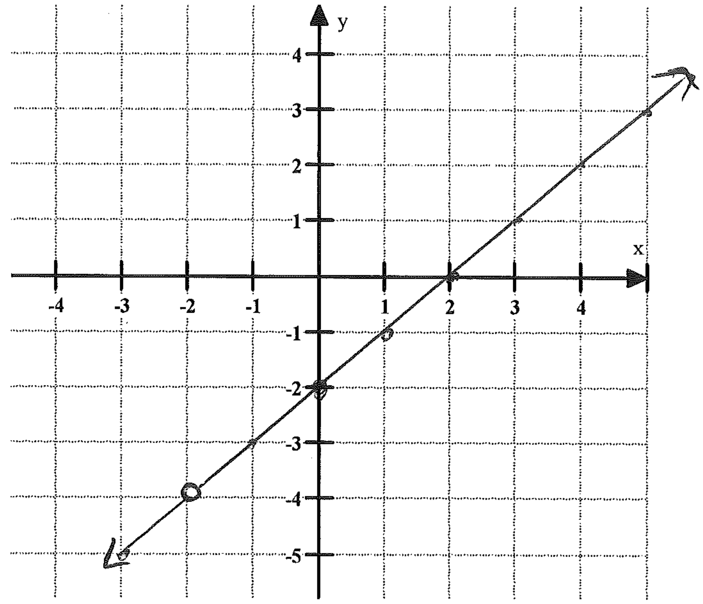
- There are two types of special cases we will look at when graphing rational expressions
- The first is a **discontinuity**: this occurs when there is a *"break"* in the graph. This break can appear as a **hole** in the function

EX 1 – Sketch the graph of $f(x)=\dfrac{x^2-4}{x+2}$:

$= \dfrac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$

$= x-2$

$\therefore x=-2$
$y=-2-2$
$=-4$ } HOLE in the graph.



EX 2 – Sketch the graph of $f(x)=\dfrac{2x^2-5x-3}{x^2-x-6}$

$= \dfrac{(2x+1)\cancel{(x-3)}}{(x+2)\cancel{(x-3)}}$

HOLE @ $(3, 7/5)$

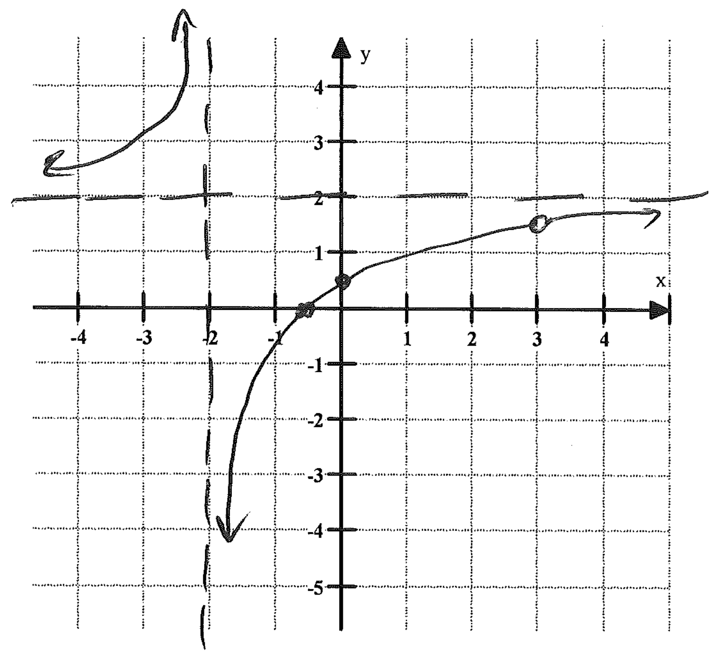
$f(x) = \dfrac{2x+1}{x+2}$, $x \neq -2, 3$.
-2 ↓ VA; 3 ↳ HOLE

VA: $x=-2$

HA: $y=2$

x-int: $x=-1/2$

y-int: $1/2$



D: $\{x \in \mathbb{R} \mid x \neq -2, 3\}$

R: $\{y \in \mathbb{R} \mid y \neq 2, 7/5\}$

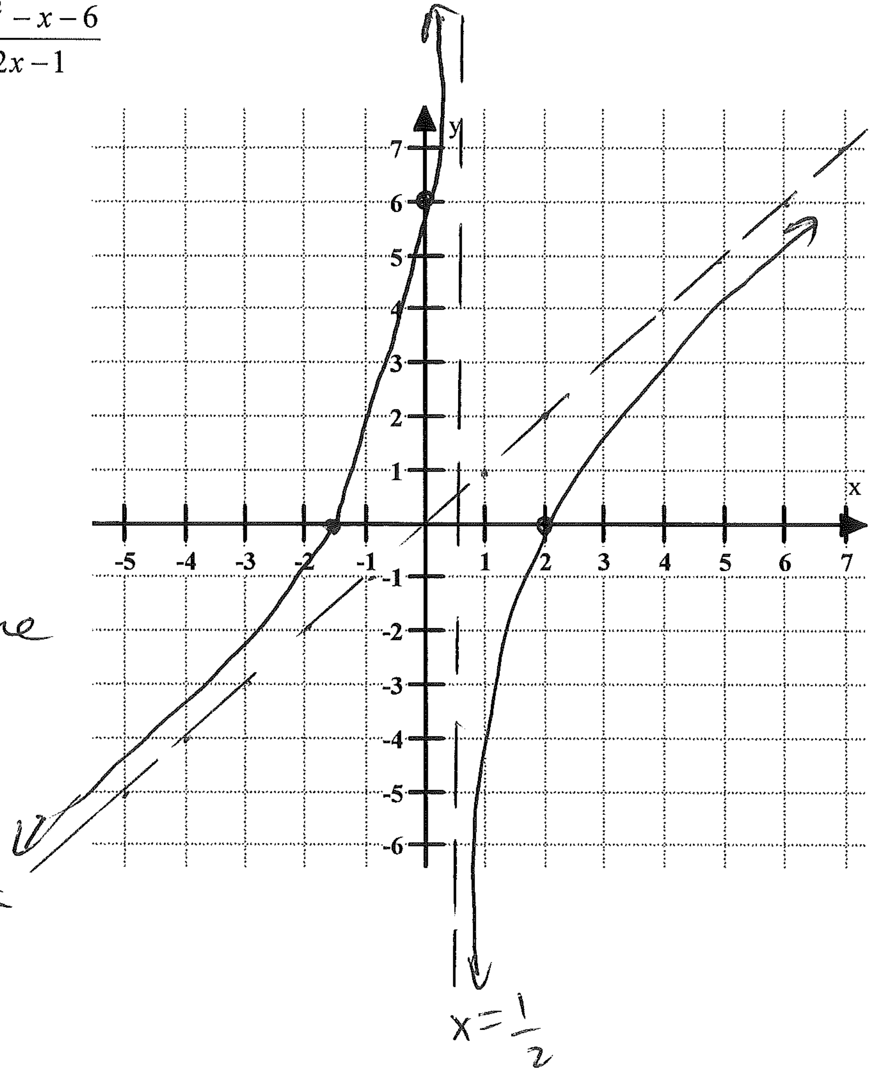
- The second special case is a different type of asymptote.
- Another asymptote other than vertical and horizontal is called a **slant** or **oblique** asymptote. **This occurs if the degree of the numerator is one greater than that of the denominator**
- A function will never have both oblique and horizontal asymptotes.
- To find the equation of the oblique asymptote, we use long division (the equation is the **quotient)**

EX 3 - Sketch the graph of $f(x)=\dfrac{2x^2-x-6}{2x-1}$

$$\begin{array}{r} x \\ 2x-1\,\big)\overline{\,2x^2-x-6\,} \\ \underline{2x^2-x\quad\;\;} \\ -6 \end{array}$$

$\therefore f(x) = x - \dfrac{6}{2x-1}$

$\therefore$ $y=x$ is the oblique asymptote.



y-int: 6

x-int: $2x^2-x-6=0$

$(2x+3)(x-2)=0$

$x=-3/2, 2$

VA: $x=1/2$

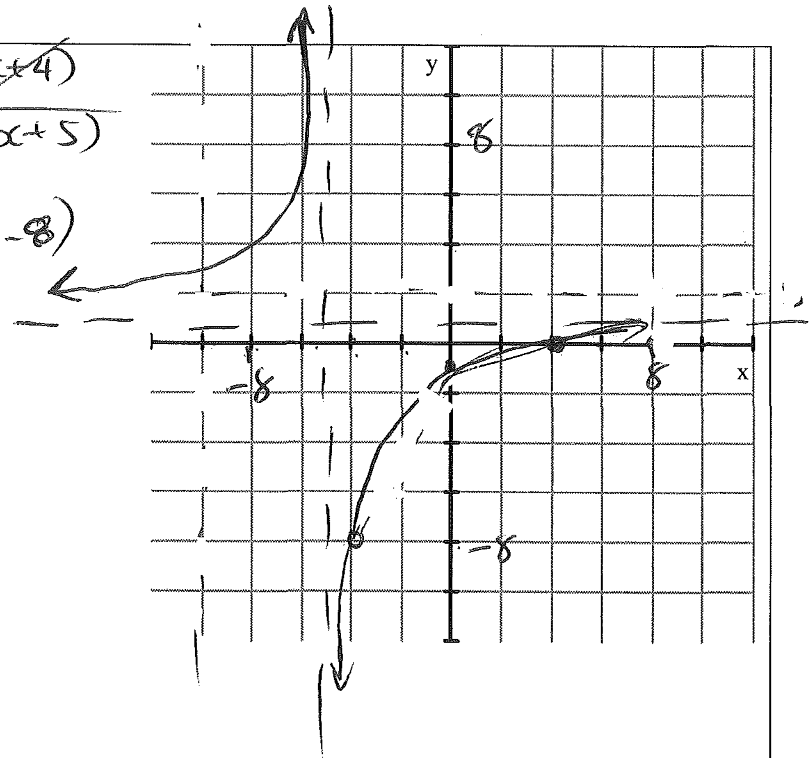
$x \to \frac{1}{2}^+ \quad y \to -\infty$

$x \to \frac{1}{2}^- \quad y \to +\infty$

Practice:

1) $f(x)=\dfrac{x^2-16}{x^2+9x+20} = \dfrac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}(x+5)}$

$= \dfrac{x-4}{x+5}$ HOLE $(-4, -8)$



HA: $y = 1$

VA: $x = -5$

x-int: 4

y-int: $-4/5$

D: $\{x \in \mathbb{R} \mid x \neq -5, -4\}$

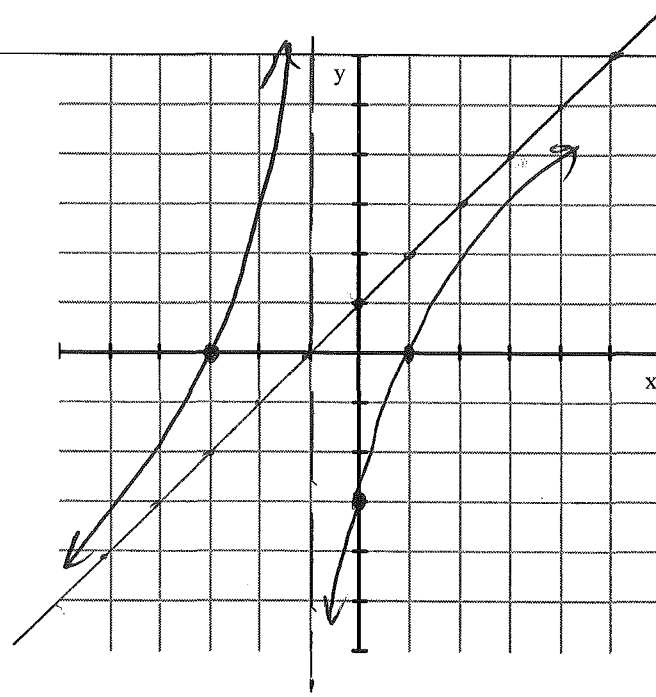
R $= \{y \in \mathbb{R} \mid y \neq 1, -8\}$

2) $f(x)=\dfrac{x^2+2x-3}{x+1} = (x+1) \quad \dfrac{-4}{x+1}$

VA: $x = -1$

oblique:

$$\begin{array}{r|l} & x + 1 \\ x+1 & x^2+2x-3 \\ & \underline{x^2+x} \\ & \quad x-3 \\ & \quad \underline{x+1} \\ & \quad\ -4 \end{array}$$



x-int: $x^2+2x-3=0$

$(x+3)(x-1)=0$

$x = -3, 1$

y-int: -3