ARITHMETIC SERIES

An arithmetic series is the indicated sum of the terms of an arithmetic sequence.

For example, 4, 9, 14, 19,... is an arithmetic sequence while 4 + 9 + 14 + 19 +... is an arithmetic series.

THE GENERAL FORMULA

$S_n = n\left(\frac{a+t_n}{2}\right)$ if the first and the last term are known OR

$S_n = \frac{n}{2}[2a+(n-1)d]$ if the first term and the common difference are known.

Where ***a*** represents the first term

n represents the number of terms

d represents the common difference

t_n represents the general (last) term

S_n represents the sum of the first n terms

EXAMPLE 1 – For the arithmetic series with $a = 2$ and $d = 4$, determine each of the following :

a) The 10th term

$t_n = a + (n-1)d$

$t_{10} = 2 + 9(4)$

$t_{10} = 38$

b) the sum of the first 10 terms.

① $S_n = n\left(\frac{a+t_n}{2}\right)$

$S_{10} = 10\left(\frac{2+38}{2}\right)$

$S_{10} = 200$

or ② $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = \frac{10}{2}[2(2)+9(4)]$

$S_{10} = 200$

the sum of 10 terms is 200

ARITHMETIC SERIES *continued...*

$a=3,\ d=5$

EXAMPLE 2 – Determine the sum of the arithmetic series $3+8+13+\ldots+58$

let $t_n = 58$

$a+(n-1)d = 58$

$3+(n-1)(5) = 58$

$3+5n-5 = 58$

$5n = 60$

$n = 12$

$S_n = n\left(\frac{a+t_n}{2}\right)$

$S_{12} = 12\left(\frac{3+58}{2}\right)$

$S_{12} = 366$

∴ Sum of series is 366

EXAMPLE 3 – In an arithmetic series, $t_2 = 10$ *and* $t_5 = 31$. Find the sum of the first 16 terms.

$a+d = 10$ ① and $a+4d = 31$ ②

② − ①: $3d = 21$

$d = 7$

Sub in ①: $a = 3$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{16} = \frac{16}{2}[2(3)+15(7)]$

$S_{16} = 888$

The sum of 16 terms is 888

EXAMPLE 4 – A marching band has 8 musicians in the first row, 10 musicians in the second row, 12 musicians in the third row, and so on. If there are 12 rows, how many musicians are in the band?

$8+10+12+\ldots$

$a = 8$

$d = 2$

$n = 12$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{12} = \frac{12}{2}[2(8)+11(2)]$

$S_{12} = 228$

228 musicians in the band

EXAMPLE 5 – A construction company building a new library is required to pay a penalty of \$1000 for the first day the completion is late, \$1500 for the second day, \$2000 for the third day, and so on. If the company payed a penalty of \$115 000, how many days late was the completion of the library?

The series is $1000 + 1500 + 2000 + \ldots$

$\therefore$ arithmetic

$a = 1000$

$d = 500$

$S_n = 115\,000$

$n = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$115\,000 = \frac{n}{2}[2(1000) + (n-1)(500)]$$

$$230000 = n(2000 + 500n - 500)$$

$$230\,000 = 1500n + 500n^2$$

$$500n^2 + 1500n - 230\,000 = 0$$

$$n^2 + 3n - 460 = 0$$

$$(n + 23)(n - 20) = 0$$

$\therefore n = -23$ or $n = 20$

inadmissible

The library was completed 20 days late.