Day 4: 3.3 – Quotient of Linear Functions: $f(x)=\dfrac{ax+b}{cx+d}$

This section will be looking at rational functions with linear expressions in **both** the numerator and the denominator. Because there is a variable in both the numerator and denominator, this will affect both the vertical and horizontal asymptotes, as well as the domain and range.

We will graph rational functions of the form $f(x)=\dfrac{ax+b}{cx+d}$ using the same steps as in 3.2:

a. Determine the asymptotes
b. Determine the x & y intercepts
c. Determine where the function is positive or negative (*lies above or below the x-axis*) by using an interval table

EX 1 –Determine the key features of the following function, and use the key features to graph the function. $f(x)=\dfrac{x+4}{x-2}$

a. Determine the asymptotes

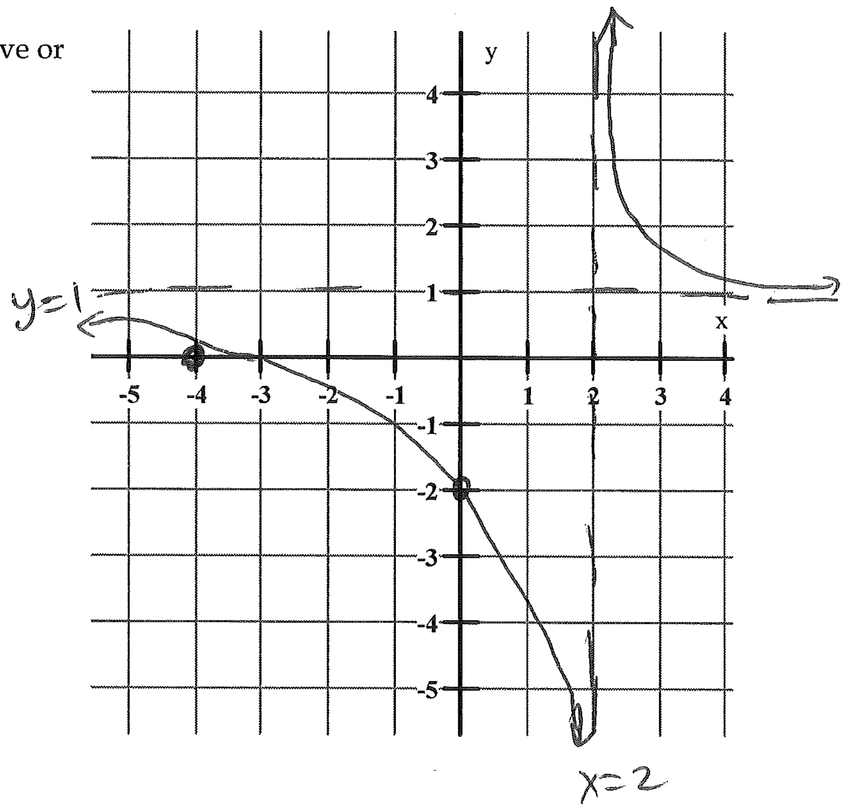
b. Determine the intercepts

a. Determine the asymptotes	b. Determine the intercepts
V.A.: $x=2$ $x\to 2^+ \quad y\to +\infty$ $x\to 2^- \quad y\to -\infty$ H.A.: $y=1$ $x\to\infty \quad y\to 1^+$ or 1 above from $x\to -\infty \quad y\to 1^-$ or 1 from below.	x-intercept(s): Set $y=0$ Solve for x $0=\dfrac{x+4}{x-2} \Rightarrow x=-4$ y-intercept: sub $x=0$ $y=\dfrac{0+4}{0-2}=-2$

c. Determine where the function is positive or negative by using an interval table

Positive: $x \in (-\infty, -4) \cup (2, \infty)$

Negative: $x \in (-4, 2)$



EX 2 –Determine the key features of the following function, and use the key features to graph the function. $f(x) = \dfrac{2x-1}{x+1}$

x-int: $x = \frac{1}{2}$

y-int: $y = \dfrac{2(0)-1}{0+1} = -1$

HA: $y = 2$

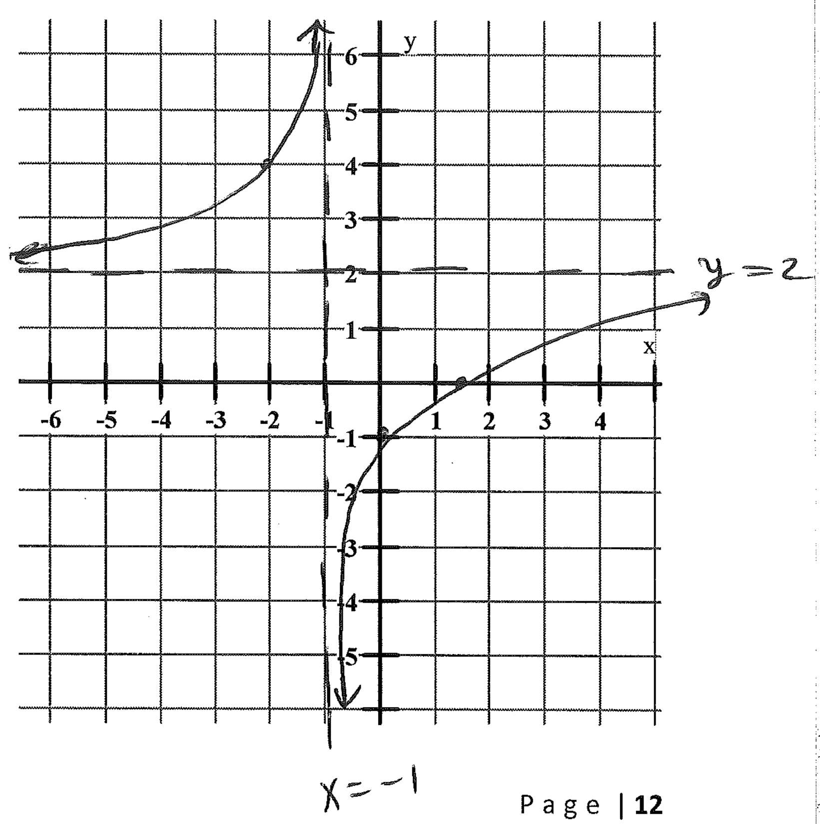
$x \to \infty \quad y \to 2^-$

$x \to -\infty \quad y \to 2^+$

VA: $x = -1$

$x \to -1^+ \quad y \to -\infty$

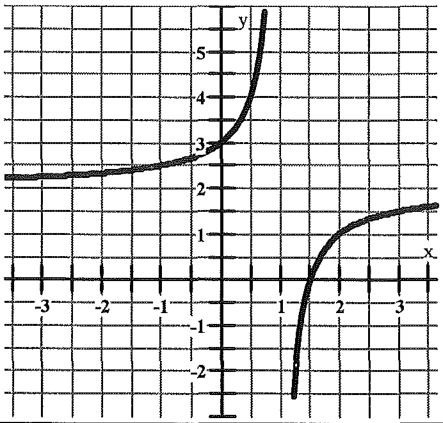
$x \to -1^- \quad y \to +\infty$



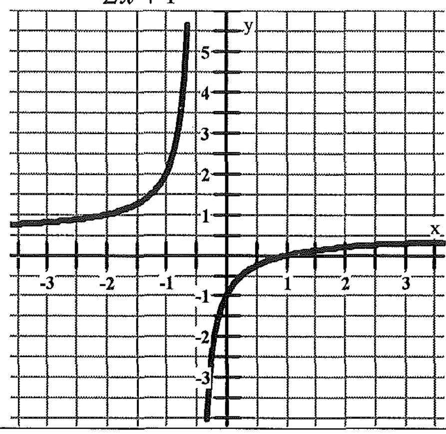
EX 3 -Compare the following graphs.

Make conclusions on the effects of a, b, c, & d on rational functions of the form $f(x)=\dfrac{ax+b}{cx+d}$**:**

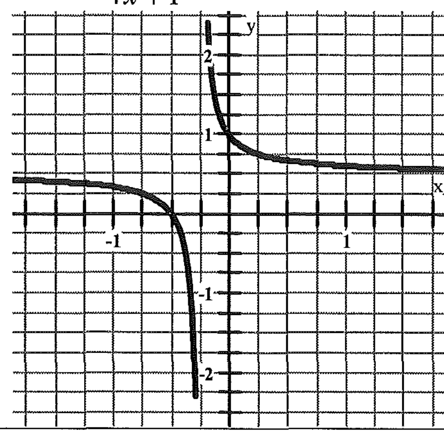
$g(x)=\dfrac{2x-3}{x-1}$



$h(x)=\dfrac{x-1}{2x+1}$



$j(x)=\dfrac{2x+1}{4x+1}$



Compare using key features: intercepts & asymptotes.

HA: $y=2$
VA: $x=1$
x-int: $\frac{3}{2}$
y-int: 3

HA: $y=\frac{1}{2}$
VA: $x=-\frac{1}{2}$
x-int: 1
y-int: -1

HA: $y=\frac{1}{2}$
VA: $x=-\frac{1}{4}$
x-int: $-\frac{1}{2}$
y-int: 1

In conclusion, a rational function of the form $f(x)=\dfrac{ax+b}{cx+d}$ has the following key features:

The equation for the **vertical asymptote** is: $x=\frac{-d}{c}$ The equation for the **horizontal asymptote** is: $y=\frac{a}{c}$

The **x-intercept** is: $-\frac{b}{a}$

The **y-intercept** is: $\frac{b}{d}$.

Sketching Rational Functions – Practice

Sketch all functions below. Show all steps as in the lessons.

Graphing Reminders:
1. Factor & simplify if necessary
2. Find the equation of any asymptotes (vertical, horizontal)
3. Find x & y intercepts
4. Use an interval table to determine where the function is positive/negative.
   * If it is a reciprocal of a quadratic, find the local max/min

a) $f(x) = \dfrac{-6}{2x+1}$

VA: $x = -\frac{1}{2}$

$x \to -\frac{1}{2}^+$ $y \to$

$x \to -\frac{1}{2}^-$ $y \to$

HA: $y = 0$

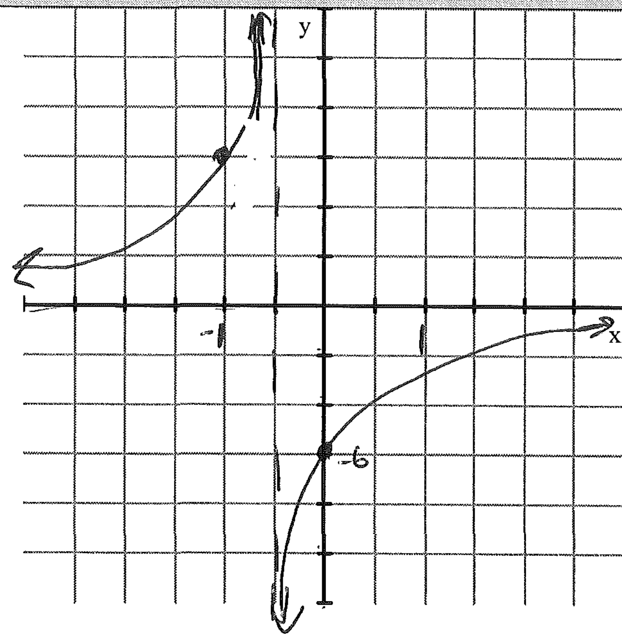
$x \to \infty$ $y \to 0^-$

$x \to -\infty$ $y \to 0^+$

x-int: none

y-int: -6

$f(x)$: $+$ | $-$ at $-\frac{1}{2}$



b) $f(x) = \dfrac{2}{x^2+2x-8} = \dfrac{2}{(x+4)(x-2)}$

VA: $x = -4$

$x \to -4^+$ $y \to -\infty$

$x \to -4^-$ $y \to +\infty$

$x = 2$

$x \to 2^+$ $y \to \infty$

$x \to 2^-$ $y \to -\infty$

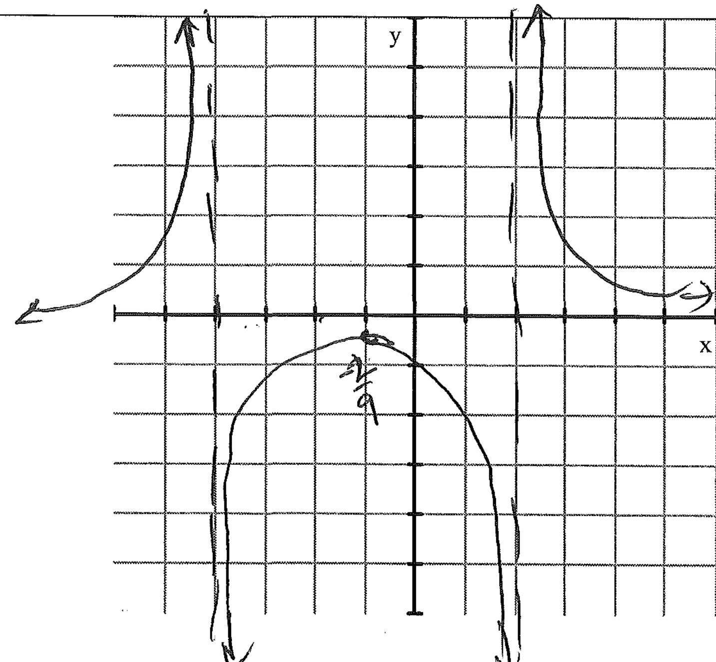
HA: $y = 0$

$x \to \infty$ $y \to 0^+$

$x \to -\infty$ $y \to 0^+$

x-int: NONE

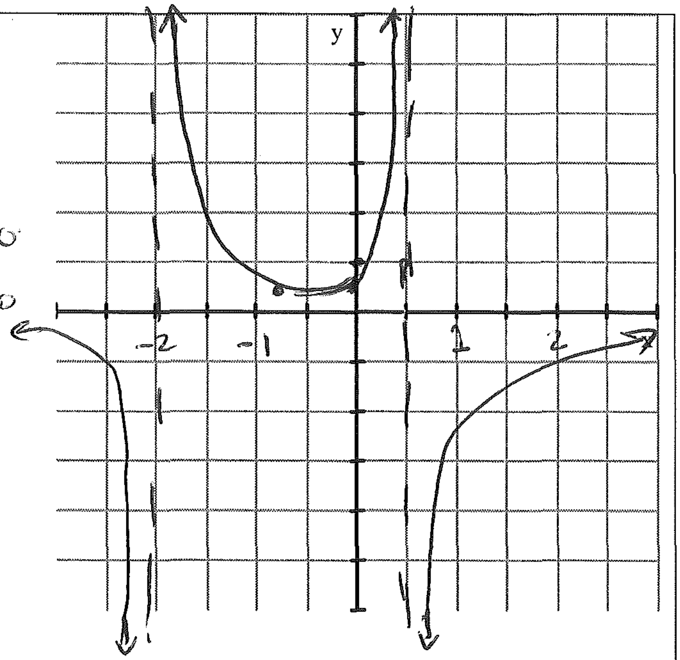
y-int: $\frac{2}{-8} = -\frac{1}{4}$



max: $x = \dfrac{-4+2}{2} = -1$

$y = \dfrac{2}{1-2-8} = \dfrac{2}{-9}$

c) $f(x) = \dfrac{-1}{2x^2+3x-2} = \dfrac{-1}{(2x-1)(x+2)}$



VA: $x = \frac{1}{2}$

$x \to \frac{1}{2}^+ \quad y \to -\infty$

$x \to \frac{1}{2}^- \quad y \to +\infty$

$x = -2$

$x \to -2^+ \quad y \to +\infty$

$x \to -2^- \quad y \to -\infty$

HA: $y = 0$

$x \to \infty \quad y \to 0^-$

$x \to -\infty \quad y \to 0^-$

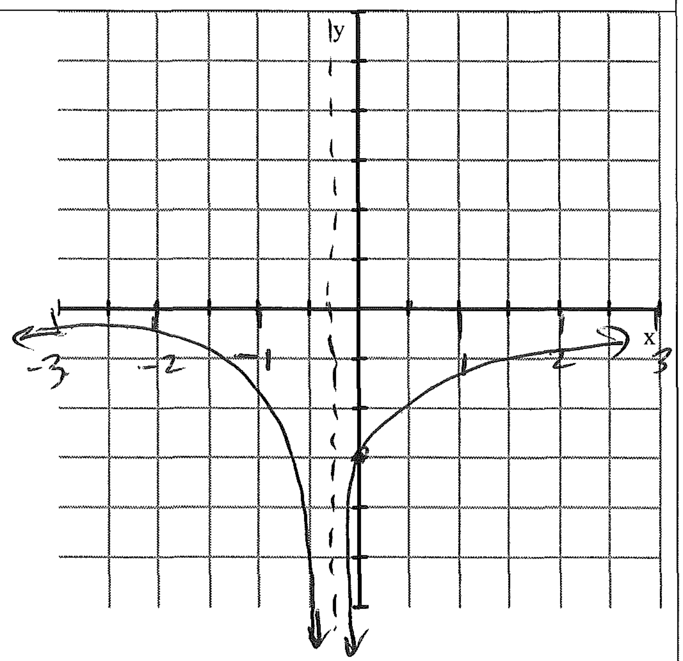
x-int: NONE

y-int: $\frac{1}{2}$

MIN: $x = \dfrac{\frac{1}{2}+(-2)}{2} = \dfrac{\frac{-3}{2}}{2} = -\dfrac{3}{4}$

$y = 0.32$

d) $f(x) = \dfrac{-3}{16x^2+8x+1} = \dfrac{-3}{(4x+1)^2}$



VA: $x = -\frac{1}{4}$

$x \to -\frac{1}{4}^+ \quad y \to -\infty$

$x \to -\frac{1}{4}^- \quad y \to -\infty$

HA: $y = 0$

$x \to \infty \quad y \to 0^-$

$x \to -\infty \quad y \to 0^-$

x-int: NONE

y-int: -3

e) $f(x)=\dfrac{x+6}{3x-1}$

HA: $y=1/3$

$x \to \infty \quad y \to \frac{1}{3}^{+}$

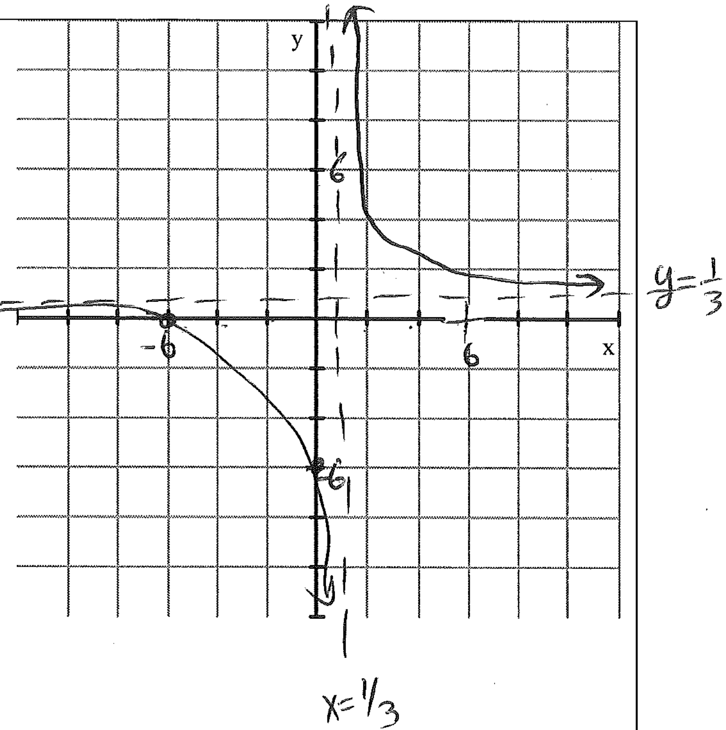
$x \to -\infty \quad y \to \frac{1}{3}^{-}$

VA: $x = 1/3$

$x \to \frac{1}{3}^{+} \quad y \to +\infty$

$x \to \frac{1}{3}^{-} \quad y \to -\infty$

y-int: -6 x-int: -6



f) $f(x)=\dfrac{2x+1}{1-4x}$

HA: $y=-\frac{1}{2}$

$x \to \infty \quad y \to -1/2^{-}$

$x \to -\infty \quad y \to -1/2^{+}$

VA: $x=\frac{1}{4}$

$x \to \frac{1}{4}^{+} \quad y \to -\infty$

$x \to \frac{1}{4}^{-} \quad y \to +\infty$

x-int: $-1/2$

y-int: 1