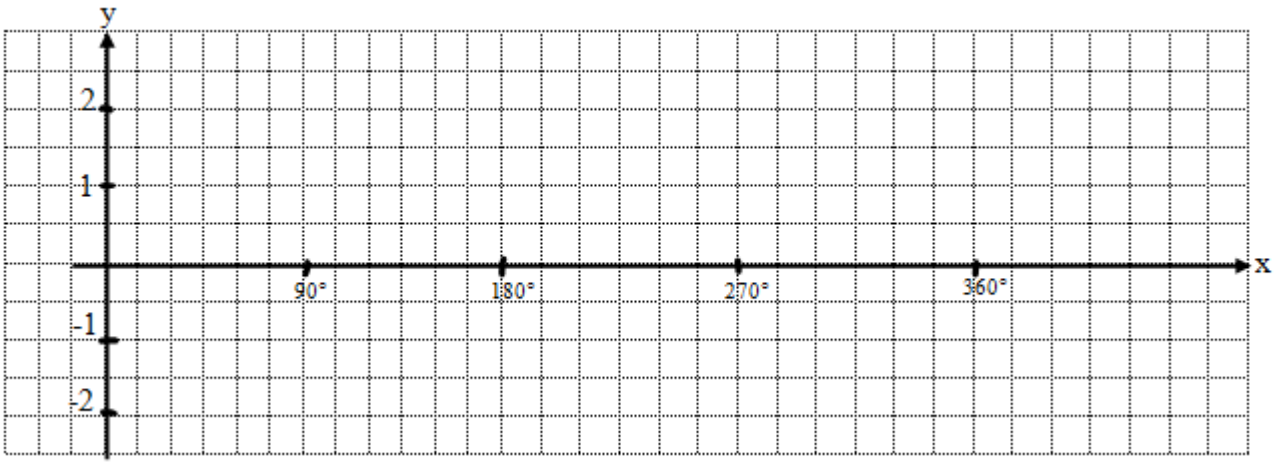
Stretches of Sinusoidal Functions

$$f(x) = asin[k(x-d)] + c \text{ and } f(x) = acos[k(x-d)] + c$$

Vertical Stretches: Investigating for *a*

Recall: $y = af(x)$ is the image of $y = f(x)$ under a transformation which causes a **vertical stretch**.
Example 1: Graph $y = \sin\theta$ and $y = 2\sin\theta$, for $0° \leq \theta \leq 360°$.

θ	0°	90°	180°	270°	360°
$y = \sin\theta$					
$y = 2\sin\theta$					



For $y = 2\sin\theta$,
1. What coordinate is affected?

2. What points are unaffected (invariant)?

3. What is amplitude, a, of the function?

4. What is the period?

5. What is the equation of the axis of the curve?

6. State the domain and range.

Example 2: Graph $y = \frac{1}{2}\sin\theta$, for $0° \leq \theta \leq 360°$ on the above grid.

θ	0°	90°	180°	270°	360°
$y = \frac{1}{2}\sin\theta$					

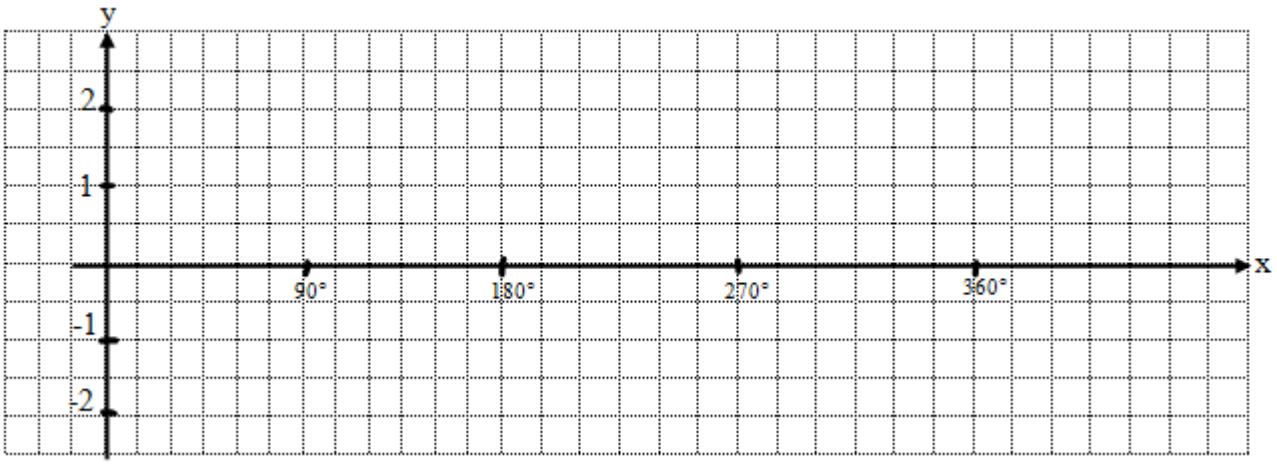
SUMMARY,
For $a > 1$, the graph is **stretched** vertically (expanded) by a factor of *a*.
For $0 < a < 1$, the graph is **compressed** vertically by a factor of *a*.

The amplitude of each function $y = a\sin\theta$ and $y = a\cos\theta$ is *a*.

Horizontal Stretches: Investigating for *k*

Recall: $y = f(\mathbf{k}x)$ is the image of $y = f(x)$ under a transformation which a causes a **horizontal stretch.**
Mapping: $(x, y) \rightarrow$

Example 1: Graph one cycle of $y = \sin\theta$ and $y = \sin 2\theta$ on the grid below using mapping notation.



For $y = \sin 2\theta$,

1. What coordinate is affected?

2. What points are unaffected (invariant)?

3. What is the amplitude, *a*, of the function?

4. What is the period?

5. What is the equation of the axis of the curve?

SUMMARY,
Recall: x says something yet does the exact opposite.
for $k > 1$, the graph is horizontally compressed by a factor of $1/k$
for $0 < k < 1$, the graph is horizontally stretched (expanded) by a factor of $1/k$

The value of *k* determines the number of degrees in the period of the graph. To determine the period of the trigonometric function, divide the period of the base curve by *k*.

$y = \sin 2\theta$ has period $\frac{360}{k}$ $\qquad$ $y = \cos 2\theta$ has period $\frac{360}{k}$

e.g. $y = \sin 2\theta$ has period $\frac{360}{2} = 180$

Ex2: $y = \sin 3\theta$ has period:

Ex3: $y = \sin \frac{1}{3}\theta$ has period:

Ex4: Determine the equation of the sine function with amplitude 4 and period 45^o. State the domain and range of one cycle.

Ex5: Sketch one cycle of $y = 3\cos \frac{1}{2}\theta$. State the amplitude, period, domain, and range of one cycle of the function.