

PRINCIPAL ACUTE ANGLE

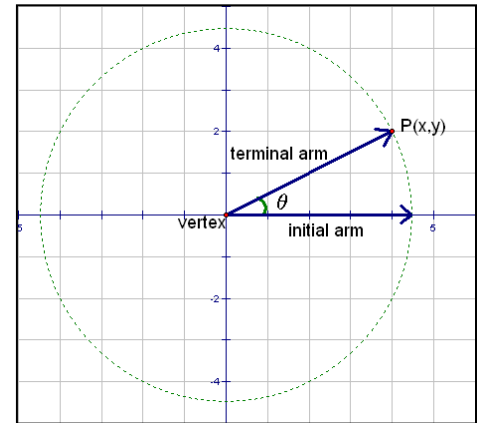
An angle is in **standard position** in a rectangular Cartesian coordinate system if:

- the vertex is at the **ORIGIN**
- the initial arm is on the **POSITIVE SIDE OF X AXIS**

The measure of angle θ is the amount of rotation from the initial arm to the terminal arm. One complete rotation is equal to **360°**.

The angle has a positive measure ($\theta \geq 0^\circ$) if the terminal arm rotates in a **COUNTER CLOCKWISE** direction.

The angle has a negative measure ($\theta < 0^\circ$) if the terminal arm rotates in a **CLOCKWISE** direction.

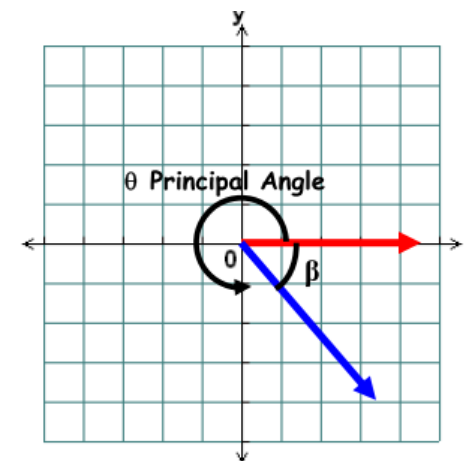
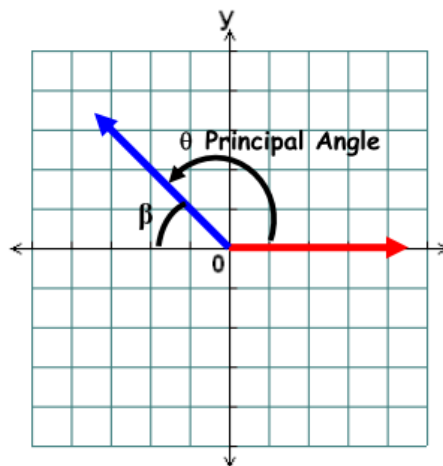
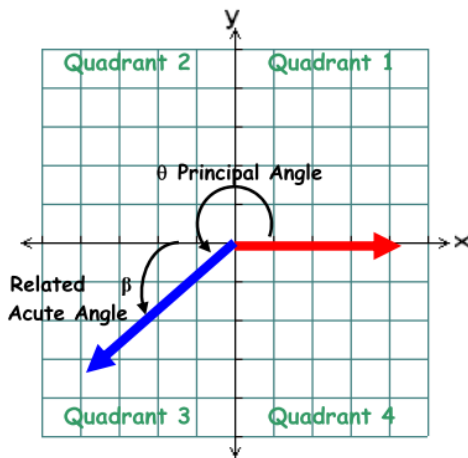


PRINCIPAL OBTUSE ANGLE

It is the counter clockwise angle between the initial arm and the terminal arm of an angle in standard position. Its value is between 0° and 360°.

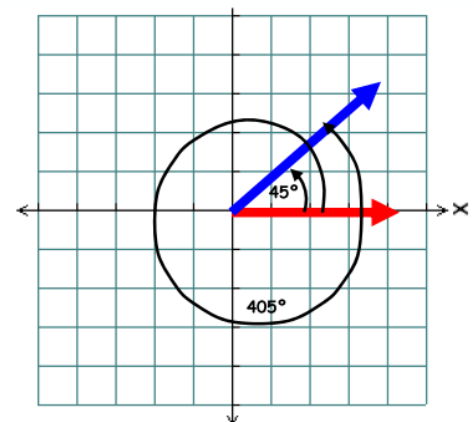
RELATED ACUTE ANGLE

It is the angle between the terminal arm of an angle in standard position and the **X - AXIS** when the terminal arm lies in quadrants 2, 3 or 4. The related acute angle is **always positive**.



COTERMINAL ANGLE

These are the angles that share the same terminal and initial arm. If you draw 45° and 405°, they will have the same terminal arm.



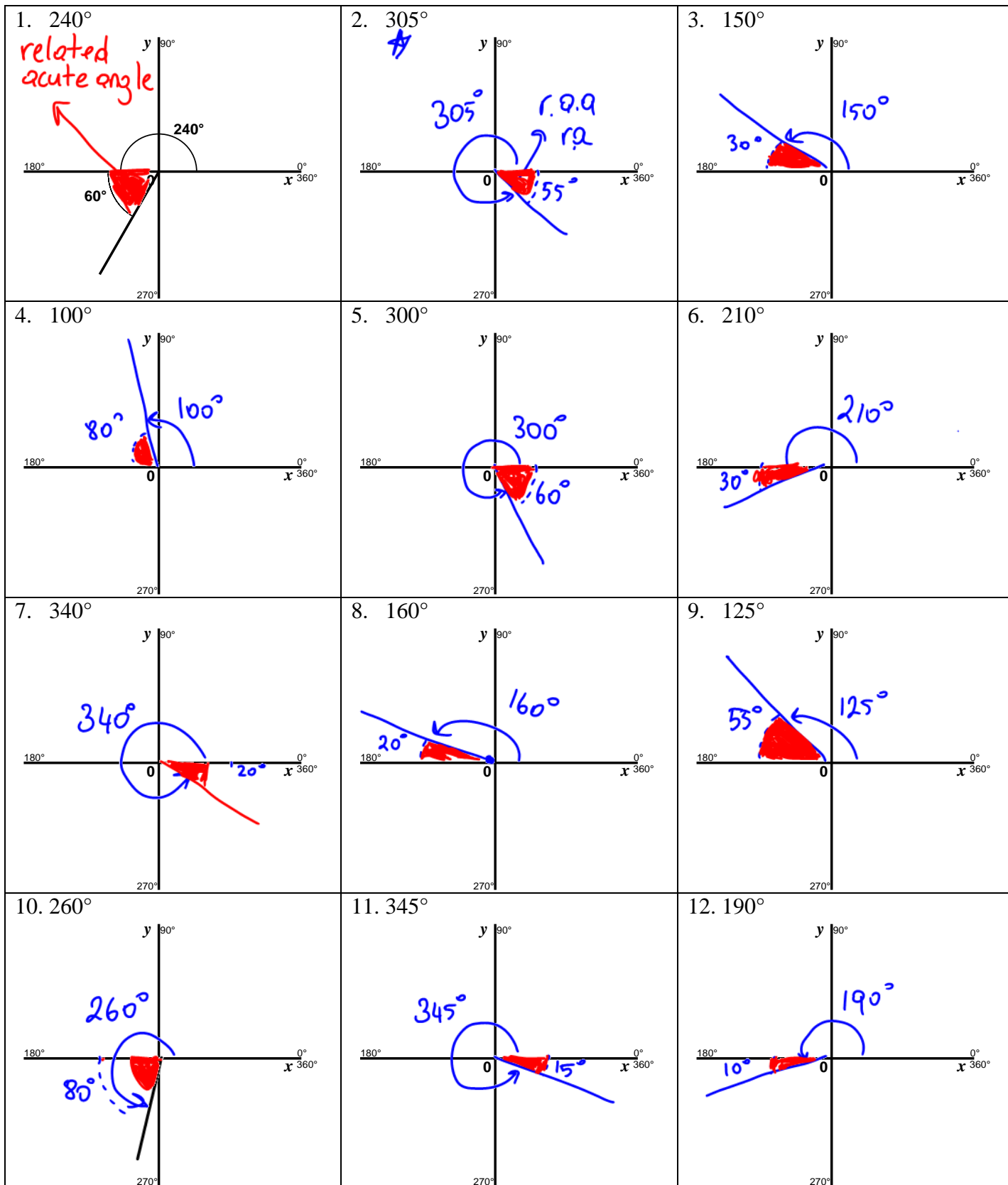
PRINCIPAL ANGLE (ACUTE/ OBTUSE) PRACTICE

| | | |
|--|--|--|
| <p>1. <i>Standard Position of an Angle</i></p> <p>$\theta = 180 - 20 = 160^\circ$</p> | <p>2.</p> <p>$180 + 35$ Principal θ 215°</p> <p>$\theta = \underline{\hspace{2cm}}$</p> <p>Related Angle 35°</p> | <p>3.</p> <p>$360 - 70$ Principal angle 290</p> <p>$\theta = \underline{\hspace{2cm}}$</p> <p>70° Related θ 70°</p> |
| <p>4.</p> <p>$180 - 78$ Principal θ 102</p> <p>$\theta = \underline{\hspace{2cm}}$</p> <p>Related θ 78°</p> | <p>5.</p> <p>$360 - 14$ $= 346$</p> <p>Principal θ $\theta = \underline{346^\circ}$</p> <p>related angle 14°</p> | <p>6.</p> <p>$180 + 87 = 267$ Principal angle 267°</p> <p>$\theta = \underline{267^\circ}$</p> <p>Related angle 87°</p> |
| <p>7.</p> <p>$180 - 87^\circ$ Principal angle 93°</p> <p>$\theta = \underline{93^\circ}$</p> <p>Related angle 87°</p> | <p>8.</p> <p>Principal angle $\theta = \underline{90^\circ}$</p> <p>Related angle 90°</p> | <p>9.</p> <p>Principal angle $\theta = \underline{270^\circ}$</p> <p>Related angle 90°</p> |
| <p>10.</p> <p>Principal angle $180 - 67$</p> <p>$\theta = \underline{113^\circ}$</p> <p>Related angle 67°</p> | <p>11.</p> <p>Principal angle $180 + 49 = 229$</p> <p>$\theta = \underline{229^\circ}$</p> <p>Related angle 49°</p> | <p>12.</p> <p>Principal angle $360 - 45^\circ$</p> <p>$\theta = \underline{315^\circ}$</p> <p>Related 45°</p> |

Answers: 1. 160° 2. 215° 3. 290° 4. 102° 5. 346° 6. 267° 7. 93° 8. 90° 9. 270° 10. 113° 11. 229° 12. 315°

RELATED ACUTE ANGLE PRACTICE

For each given angle, draw the angle *as a solid arc* as shown in the example for #1. For each diagram, also draw *as a dotted arc* and label the angle between the terminal arm and the x-axis.



COTERMINAL ANGLE PRACTICE

| | | |
|--|---|---|
| <p>1.</p> <p>Quadrant 2 Quadrant 1 160° $160 + 360$ 20° 520 $\theta = \underline{\hspace{2cm}}$</p> <p>Quadrant 3 Quadrant 4</p> | <p>2.</p> <p>215° $215 + 360$ 35° 575 $\theta = \underline{\hspace{2cm}}$</p> | <p>3.</p> <p>215 $215 + 360 + 360$ 35° 935° $\theta = \underline{\hspace{2cm}}$</p> |
| <p>4.</p> <p>$-145 - 360$ -505° 35° $\theta = \underline{\hspace{2cm}}$ -145</p> | <p>5.</p> <p>$-145 - 360 - 360$ -865° 35° $\theta = \underline{\hspace{2cm}}$ -145</p> | <p>6.</p> <p>$-70 - 360 - 360$ -790° 70° $\theta = \underline{\hspace{2cm}}$ -70</p> |
| <p>7.</p> <p>$78 - 360 = -282$ $-282 - 360$ -642 78° $\theta = \underline{\hspace{2cm}}$ -282</p> | <p>8.</p> <p>$90 + 3(360)$ 90° $\theta = \underline{\hspace{2cm}}$ 1170° $360 + 360 + 360$ $+ 90 = 1170^\circ$</p> | <p>9.</p> <p>$-270 - 2(360)$ -990° -270 $\theta = \underline{\hspace{2cm}}$</p> |

10. In each case below, find the positive or negative angle which is coterminal to the one given.

| # | Original Angle | Additional Rotations | Coterminal Angle | # | Original Angle | Additional Rotations | Coterminal Angle |
|---|----------------|----------------------|---------------------------|---|----------------|----------------------|-------------------------------|
| a | 60° | +2 | $60 + 2(360) = 780^\circ$ | g | -25° | +2 | $-25 + 2(360) = 695^\circ$ |
| b | 130° | +3 | $130 + 3(360) = 1210$ | h | -1285° | +3 | $-1285 + 3(360) = -205^\circ$ |
| c | 200° | +1 | $200 + 360 = 560$ | i | -157° | -2 | $-157 - 2(360) = -877^\circ$ |
| d | 310° | +4 | $310 + 4(360) = 1750$ | j | 255° | -3 | $255 - 3(360) = -825^\circ$ |
| e | 42° | +2 | $42 + 2(360) = 762$ | k | 800° | -5 | $800 - 5(360) = -100$ |
| f | 734° | -2 | $734 - 2(360) = 14$ | l | -1440° | +4 | $-1440 + 4(360) = 0$ |

11. Given the angle $0^\circ \leq \theta < 360^\circ$ which is *coterminal* to each of the following angles. Include one middle step, then give the quadrant of the angle.

angle
coterminal $\theta = 500 - 360 = 140$ (Q2)
 $\theta = 670 - 360 = 310$ (Q4)
 $\theta = 415 - 360 = 55$ (Q1)
 $\theta = 905 - 720 = 185$ (Q3)
 $\theta = 2000 - 5(360) = 200$ (Q3)
 $\theta = 1234 - 3(360) = 154$ (Q2)

12. Give the angle $0^\circ \leq \theta < 360^\circ$ which is *coterminal* to each of the following angles. Include one middle step, then give the quadrant of the angle.

60
 $\theta = -70 + 360 = 290$ (Q4)
 $\theta = -200 + 360 = 160$ (Q2)
 $\theta = -400 + 2(360) = 320$ (Q4)
 $\theta = -700 + 2(360) = 20$ (Q1)
 $\theta = -820 + 3(360) = 260$ (Q3)
 $\theta = -2000 + 6(360) = 160$ (Q2)

13. After each given angle, write the quadrant number in the brackets, then state the first quadrant angle ($0^\circ \leq \alpha < 90^\circ$) which is the *related angle* to each of the following angles.

a) 125° (Q2) $\alpha = 180 - 125 = 55^\circ$
 b) 169° (Q2) $\alpha = 180 - 169 = 11^\circ$
 c) 241° (Q3) $\alpha = 241 - 180 = 61^\circ$
 d) 318° (Q4) $\alpha = 360 - 318 = 42^\circ$
 e) 276° (Q4) $\alpha = 360 - 276 = 84^\circ$
 f) 267° (Q3) $\alpha = 267 - 180 = 87^\circ$

14. Find the angle $0^\circ \leq \theta < 360^\circ$ which is *coterminal* to each of the following angles, then the *related acute angle* ($0^\circ \leq \alpha < 90^\circ$) which matches that angle.

a) 490° $\theta = 490 - 360 = 130$ (Q2) $\alpha = 50^\circ$
 b) 685° $\theta = 685 - 360 = 325$ (Q4) $\alpha = 35^\circ$
 c) 820° $\theta = 820 - 2(360) = 100$ (Q2) $\alpha = 80^\circ$
 d) -756° $\theta = -756 + 3(360) = 324$ (Q4) $\alpha = 36^\circ$
 e) -263° $\theta = -263 + 360 = 97$ (Q2) $\alpha = 83^\circ$
 f) -2000° $\theta = -2000 + 6(360) = 160$ (Q2) $\alpha = 20^\circ$



Answers:

4. -505° 5. -865° 6. -790° 7. -642° 8. 1170° 9. -990°
 10. a) 780° b) 1210° c) 560° d) 1750° e) 762° f) 14°
 g) 695° h) -205° i) -877° j) -825° k) -1000° l) 0°
 11. a) 140° ; Q2 b) 310° ; Q4 c) 55° ; Q1 d) 185° ; Q3 e) 200° ; Q3 f) 154° ; Q2
 12. a) 290° ; Q4 b) 160° ; Q2 c) 320° ; Q4 d) 20° ; Q1 e) 260° ; Q3 f) 160° ; Q2
 13. a) Q2 ; 55° b) Q2 ; 11° c) Q3 ; 61° d) Q4 ; 42° e) Q3 ; 84° f) Q2 ; 26°
 14. a) 50° b) 35° c) 80° d) 36° e) 83° f) 20°