Recursive Formulas

So far, we have learned to find the general term of an arithmetic or geometric sequence using...

ARITHMETIC	GEOMETRIC
$t_n = a + (n-1)d$	$t_n = ar^{n-1}$

These formulas are **EXPLICIT** in nature. That means, they are definite or clearly expressed. Using these formulas, we can find **ANY** term.

RECURSION – The process of describing the next term in a sequence by relating it to the previous term. In other words, the recursion formula will tell us how to get a term, **BUT ONLY IF** we know the previous one.

For example, in order to know the 50th term, you must know the 49th. In order to know the 49th, you must know the 48th, etc...

A RECURSION FORMULA HAS 2 COMPONENTS

1. An initial condition that provides a starting point.

   i.e. $t_1 = 8$ (start at 8)

   $t_1 = -5$ (start at –5)

2. A recursive formula that tells how to derive a term using the one that came previous.

   i.e. $t_n = t_{n-1} + 2$ means: get any term by adding 2 to the previous term

   $t_n = 3t_{n-1}$ means: get any term by multiplying previous term by 3

EXAMPLE 1 – Find the first 6 terms defined by the recursive formula : $t_1 = 5 \quad t_n = t_{n-1} + 3$

$t_1 = 5$
$t_2 = 8$
$t_3 = 11$
$t_4 = 14$
$t_5 = 17$
$t_6 = 20$

OR, the first 6 terms are:

5, 8, 11, 14, 17, 20

∴ arithmetic sequence

Recursion Formula Practise Questions

1) Write the first 5 terms of the sequence defined by $t_1 = 1$, $t_n = t_{n-1} + n$, $n > 1$.

$t_1 = 1$

$t_2 = t_1 + 2$
$= 1 + 2$
$= 3$

$t_3 = t_2 + 3$
$= 3 + 3$
$= 6$

$t_4 = t_3 + 4$
$= 6 + 4$
$= 10$

$t_5 = t_4 + 5$
$= 10 + 5$
$= 15$

2) Write a recursion formula for each sequence:

a) 4, 11, 18, 25,...

$t_1 = 4$
$t_n = t_{n-1} + 7$

b) 32, 26, 20, 14,...

$t_1 = 32$
$t_n = t_{n-1} - 6$

c) 1, -10, 100, -1000,...

$t_1 = 1$
$t_n = -10 t_{n-1}$

d) 32, 16, 8, 4,...

$t_1 = 32$
$t_n = \frac{1}{2} t_{n-1}$

e) 4, 5, 20, 100, 2000, ...

$t_1 = 4$
$t_2 = 5$
$t_n = t_{n-1} \times t_{n-2}$, $n > 2$

3) A sequence is defined by the recursion formula $t_1 = 3$, $t_n = t_{n-1} + 10$, $n > 1$. Determine t_{100}.

$a = 3$
$d = 10$ } arithmetic

$t_n = a + (n-1)d$
$\therefore t_{100} = 3 + 99(10)$ $\therefore t_{100} = 993$

4) Write the first 5 terms of the sequence defined by $t_1 = 1$, $t_2 = 2$, $t_n = t_{n-1} + t_{n-2}$, $n > 2$.

First 5 terms are 1, 2, 3, 5, 8