Are You Losing Your Trigonometric Mind???

Here is a systematic way to produce a trigonometry proof:

1. Start with the more complicated looking side.

2. Substitute in the 8 previously proven trig. identities in order to manipulate the left side and/or right side to equal one another.

    a. Convert csc, sec, cot, and tan to expressions involving only sin and cos. Remember, these can have exponents.

    b. Make the Pythagorean identity one of your favourites. Always look for $\sin^2x$ and $\cos^2x$ to make it 1, and consider replacing occurrences of $\sin^2x$ with $1 - \cos^2x$ and occurrences of $\cos^2x$ with $1 - \sin^2x$.

    c. Don't forget to use the other 2 Pythagorean identities (which can be found by dividing the original Pythagorean identity by (1) $\sin^2x$ and (2) $\cos^2x$.

    d. Remember that the Pythagorean identities only work with squared exponents.

3. Use your algebra & fraction rules.

    a. If possible, expand all the expressions in sight (distributive property/FOIL) and combine like terms simplify.

    b. If possible, factor numerator and denominator and cancel common factors if any.

    c. When adding fractions - get a common denominator (when in doubt - multiply the denominators to find the LCD).

    d. When multiplying fractions - cancel common factors from any numerator & any denominator.

    e. When dividing fractions, cancel like denominators where possible. If not - take the reciprocal of the second fraction and multiply.

Are You Having an 'IDENTITY' Crisis?

Ex1: Prove $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$

$$LS = \sin^2\theta + \cos^4\theta$$
$$\sin^2\theta + (\cos^2\theta)(\cos^2\theta)$$
$$\sin^2\theta + (\cos^2\theta)(1-\sin^2\theta)$$
$$\underbrace{\sin^2\theta + \cos^2\theta}_{1} - \sin^2\theta\cos^2\theta$$
$$1 - \sin^2\theta\cos^2\theta$$

$$RS = \cos^2\theta + \sin^2\theta\sin^2\theta$$
$$= \cos^2\theta + \sin^2\theta(1-\cos^2\theta)$$
$$= \underbrace{\cos^2\theta + \sin^2\theta} - \sin^2\theta\cos^2\theta$$
$$= 1 - \sin^2\theta\cos^2\theta$$

$LS = RS$

✓ QED

Ex2: Prove $\dfrac{\sin x}{1+\cos x} = \csc x - \cot x$

$$LS = \frac{\sin x}{1+\cos x}$$

$$RS = \csc x - \cot x$$
$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x}$$
$$= \frac{1-\cos x}{\sin x} \cdot \frac{\sin x}{\sin x}$$
$$= \frac{(1-\cos x)\sin x}{\sin^2 x}$$
$$= \frac{(1-\cos x)\sin x}{1-\cos^2 x} \longrightarrow \text{DOS}$$
$$= \frac{\cancel{(1-\cos x)}\sin x}{\cancel{(1-\cos x)}(1+\cos x)}$$
$$= \frac{\sin x}{1+\cos x}$$

$LS = RS$

QED ✓

Or

$$LS = \frac{\sin x}{1+\cos x} \cdot \frac{(1-\cos x)}{(1-\cos x)} \qquad RS = \frac{1-\cos x}{\sin x}$$
$$= \frac{\sin x(1-\cos x)}{(1+\cos x)(1-\cos x)} \leftarrow \text{DOS}$$
$$= \frac{\sin x(1-\cos x)}{1-\cos^2 x} \quad \sin^2 x$$
$$= \frac{\sin x(1-\cos x)}{\sin^2 x}$$
$$= \frac{1-\cos x}{\sin x}$$

$LS = RS$

✓ QED

Text p.310 #7, 8, 10, 11