geOMETRIC seqUENCES

A sequence where every successive term is found by MULTIPLYING BY THE SAME NUMBER is called **GEOMETRIC**

CHECK: Pick any term, divide it by the previous term. If the result is always the same no matter where in the sequence you begin, then the sequence is geometric.

EXAMPLE 1 – Is the sequence geometric?

a) 2, 6, 18, 54, 162, 486, ... $\frac{486}{162} = 3$

$\frac{18}{6} = 3$

The terms are separated by a **COMMON RATIO** of 3 (we will call it "r")

b) 12, 6, 3, 1.5, 0.75, 0.375, ... $\frac{0.375}{0.75} = \frac{1}{2}$

$\frac{3}{6} = \frac{1}{2}$

The terms are separated by a **COMMON RATIO** of $\frac{1}{2}$

EXAMPLE 2 – Find the general term of the following geometric sequence

3, −12, 48, −192, ... the common ratio is -4

a

Observe and continue the pattern... ***Symbolically...***

1st term	3	a
2nd term	3(−4)	ar
3rd term	3(−4) (−4)	ar^2
4th term	3(−4)(−4)(−4)	ar^3
5th term	$3(-4)^4$	ar^4
6th term	$3(-4)^5$	ar^5

n^{th} term *Do you see the pattern?* → ar^{n-1}

Geometric Sequences *continued...*

CONCLUSION: To find the general term of an geometric sequence

$$t_n = a r^{n-1} \quad \text{vs.} \quad \underset{\text{arithmetic}}{t_n = a + (n-1)d}$$

where a is the first term

n is the number of the term

and r is the common ratio

EXAMPLE 3 – Given the geometric sequence 3, 6, 12, 24, ...

a) Find the 14th term

$a = 3$

$r = 2$

$\therefore t_n = ar^{n-1}$

$t_n = 3(2)^{n-1}$

$t_{14} = 3(2)^{13}$

$t_{14} = 24576$

The 14th term is 24576.

b) Which term is 384?

let $t_n = 384$

$384 = 3(2)^{n-1}$

$128 = 2^{n-1}$

same base, both sides:

$2^7 = 2^{n-1}$

$\therefore n-1 = 7 \quad \therefore n = 8$

$\therefore$ the eight term is 384

EXAMPLE 4 – The 3rd term of an geometric sequence is 20 while the 6th term of the same sequence is –540. Find the general term of the sequence and state the first 6 terms.

$t_3 = 20$ and $t_6 = -540$

$t_n = ar^{n-1}$

$t_3 = ar^2 \qquad t_6 = ar^5$

$\therefore ar^2 = 20$ ① and $ar^5 = -540$ ②

② ÷ ① : $\frac{\cancel{a}r^5}{\cancel{a}r^2} = \frac{-540}{20}$

$\therefore r^3 = -27$

$\therefore \boxed{r = -3}$

now sub in ①: $a(-3)^2 = 20$

$a = \frac{20}{9}$

$\therefore t_n = \frac{20}{9}(-3)^{n-1}$

$t_n = 20(-3)^{n-3}$