# *Day 2: 3.2 Reciprocal of a Quadratic Function*

A quadratic function has the form $f(x) = ax^2 + bx + c$ in standard form. What does the graph of the reciprocal of a quadratic look like?

Vertical asymptotes occur when the denominator of a rational expression is zero. ***The roots of a quadratic expression in the denominator correspond to any vertical asymptotes.***

**Since a quadratic may have zero, one or two real roots, the reciprocal of a quadratic may have zero, one or two vertical asymptotes.**

**In general, to graph the reciprocal of a quadratic function:**

a. Factor and simplify the expression
b. Determine the VA and HA
c. Determine the x & y intercepts
d. Determine where the function is positive or negative (*lies above or below the x-axis*) using an interval table

**EX 1** – Graphing a reciprocal of a quadratic function **with two zeros:** $f(x) = \dfrac{1}{x^2 + 2x - 3} = \dfrac{1}{(x+3)(x-1)}$

a. Factor and simplify:

$f(x) = \dfrac{1}{(x+3)(x-1)}$

b. Determine the asymptotes

VA $x = -3$ or $x = 1$

(set denominator $= 0$)

HA

$x \to \infty \quad y \to 0^+$

$x \to -\infty \quad y \to 0^+$

c. Determine the intercepts

x-intercept(s): set $y = 0$

$0 = \dfrac{1}{(x+3)(x-1)}$

$1 \neq 0$ ∴ NO x-int

y-intercept:

sub $x = 0$

$y = \dfrac{1}{(0+3)(0-1)}$

$= -\dfrac{1}{3}$

d. Determine where the function is positive or negative (*lies above or below the x-axis*) by using an interval table

| | $(-\infty,-3)$ | $(-3,1)$ | $(1,\infty)$ |
|---|---|---|---|
| $f(x)$ | + | − | + |

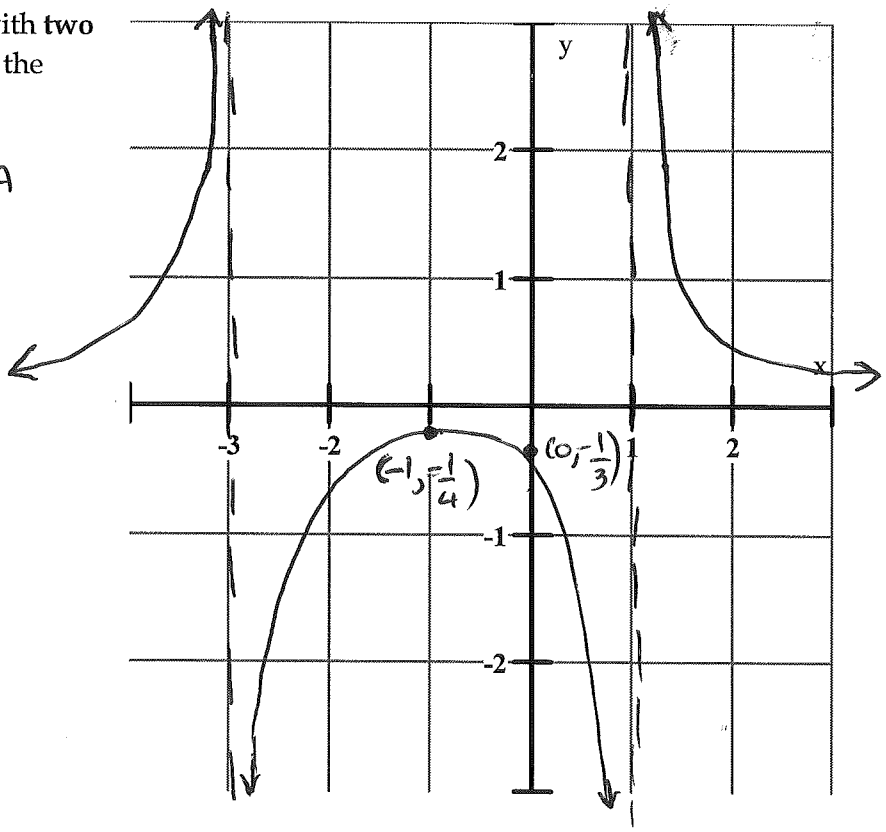
* For reciprocal of quadratic functions with **two zeros**, you also need to find the value of the local maxima/minima

Take average of VA x- coordinates

$$x = \frac{-3+1}{2} = -1$$

$$y = \frac{1}{(-1+3)(-1-1)}$$

$$= -\frac{1}{4}$$




Domain: $x \in (-\infty,-3) \cup (-3,1) \cup (1,\infty)$

OR $\{x \in \mathbb{R} \mid x \neq -3, 1\}$

Range: $y \in (-\infty,-1/4] \cup (0,\infty)$

OR $\{y \in \mathbb{R} \mid y \leq -\frac{1}{4} \text{ or } y > 0\}$

$x \to -3^+ \quad y \to -\infty$

$x \to -3^- \quad y \to +\infty$

$x \to 1^+ \quad y \to +\infty$

$x \to 1^- \quad y \to -\infty$