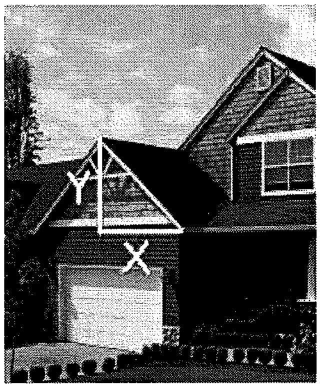
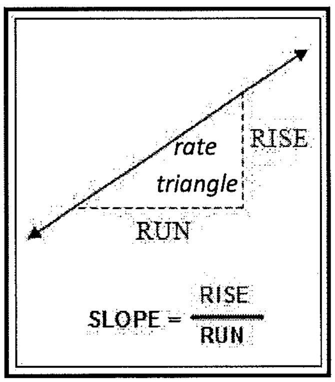
# Slope and Equations of Lines

## Slope

The word **slope** (aka: gradient, incline, pitch) is used to describe the measurement of the *steepness* of a straight line or line segment. The higher the slope, the steeper the line is. The slope of a line is a *rate of change*.




Slope is important in many real world situations. For example, a **wheelchair ramp** must be built so that its grade or steepness is small enough that a person in a wheelchair is capable of going up the ramp on his or her own. In addition, **roads** along mountainsides are designed with a small grade so that trucks do not drive out of control. If this happens, the positive slope of a mountain can assist slowing a truck down along an escape ramp.

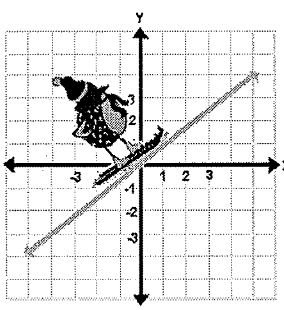


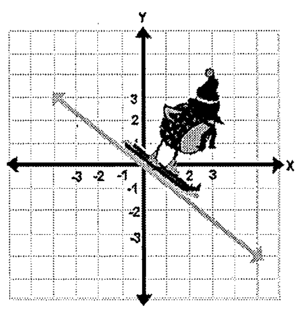

The mathematical symbol for **slope** is ***m***.

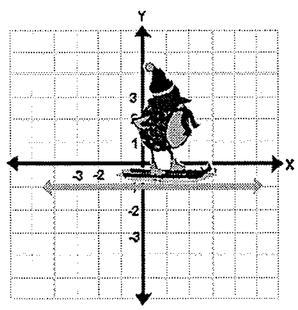
**Ex1. Find** the slope of the line that passes through $A(-3,4)$ *and* $B(5,-2)$.

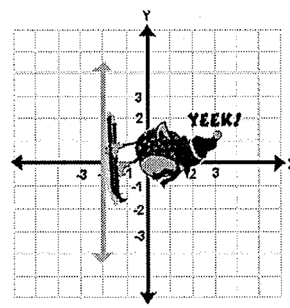
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{5-(-3)}$$

$$= \frac{-6}{8} = \frac{-3}{4}$$




slope is 1 (positive) slope is -1 (negative) slope is 0 (horizontal) slope is undefined (vertical)

## Equation of the line:

Two Formats You Can Start With:

$y = mx + b$ *(slope/y-intercept form)*

$m =$ slope

$b =$ y-intercept

$y = m(x - p) + q$ *(slope/point form)*

$m =$ slope

$(p, q) =$ $(x_1, y_1)$ a point.

End With Either:

$y = mx + b$ *(slope/y-intercept form)* *or* $Ax + By + C = 0$ *(standard form)*

**a)** Find the equation of a line in **Standard Form** given a slope of -6 passing through the point $R(-2,3)$.

$y = m(x - x_1) + y_1$

$y = -6(x - (-2)) + 3$

$= -6(x+2) + 3$

$= -6x - 12 + 3$

$\boxed{y = -6x - 9}$

standard: $6x + y + 9 = 0$

**b)** Find the equation of the line in **Standard Form** passing through $K(-2,5)$ *and* $G(6,-1)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-5}{6-(-2)}$

$= \frac{-6}{8}$

$= -\frac{3}{4}$

$y = m(x - x_1) + y_1$ use $(6,-1)$

$y = -\frac{3}{4}(x-6) - 1$

$y = -\frac{3}{4}x + \frac{18}{4} - 1$

$y = -\frac{3}{4}x + \frac{9}{2} - 1$

$\boxed{y = -\frac{3}{4}x + \frac{7}{2}}$

$4y = -3x + 14$

$3x + 4y - 14 = 0$

standard form

c) Find the equation of the line in slope/y_intercept form given a slope of $\frac{2}{3}$ passing through $P(-4,5)$.

$y = m(x - x_1) + y_1$

$y = \frac{2}{3}(x+4) + 5$

$= \frac{2}{3}x + \frac{8}{3} + 5$

$= \frac{2}{3}x + \frac{8}{3} + \frac{15}{3}$

$\boxed{y = \frac{2}{3}x + \frac{23}{3}}$

d) Find the equation of the line in y=mx+b that is perpendicular to y =3x+5 passing through W (-2, 4).

$m = 3 \quad \perp$ slope $= \frac{-1}{3}$

$y = \frac{-1}{3}(x+2) + 4$

$y = \frac{-1}{3}x - \frac{2}{3} + 4$

$= \frac{-1}{3}x - \frac{2}{3} + \frac{12}{3} \Rightarrow y = \frac{-1}{3}x + \frac{10}{3}$

e) slope is undefined, passes through (4, -3).

$\checkmark$

vertical line

$\therefore x = 4$

f) Horizontal line passing through (-4, -2).

$\checkmark$

horizontal line

$\therefore y = -2$

# Midpoint Mania

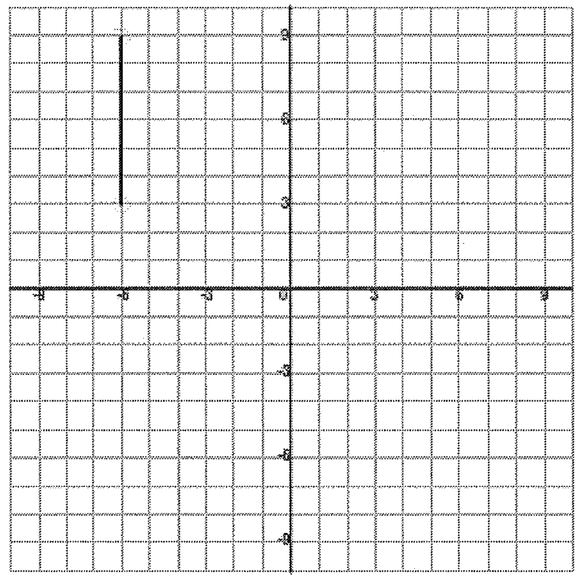
## Task 1: The Midpoint Formula

### VERTICAL LINE SEGMENTS

What is the midpoint of the line segment AB?

A (-6, 9)
B (-6, 3)

M ( $\underline{-6}$ , $\underline{6}$ )



How can the midpoint be determined using a mathematical calculation instead of counting the number of squares?

Answer:

Find halfway of 3 and 9

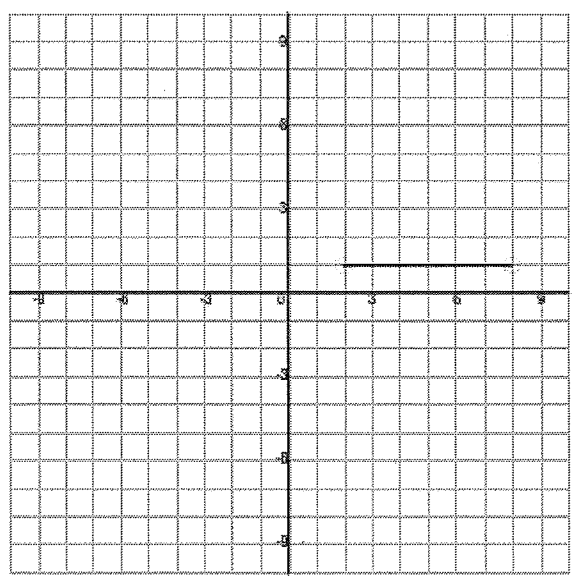
which is at $\frac{3+9}{2} = 6$

### HORIZONTAL LINE SEGMENTS

What is the midpoint of the line segment AB?

A (2, 1)
B (8, 1)

M ( $\underline{5}$ , $\underline{1}$ )



How can the midpoint be determined using a mathematical calculation instead of counting the number of squares?

Answer:

Find the average of 2, 8

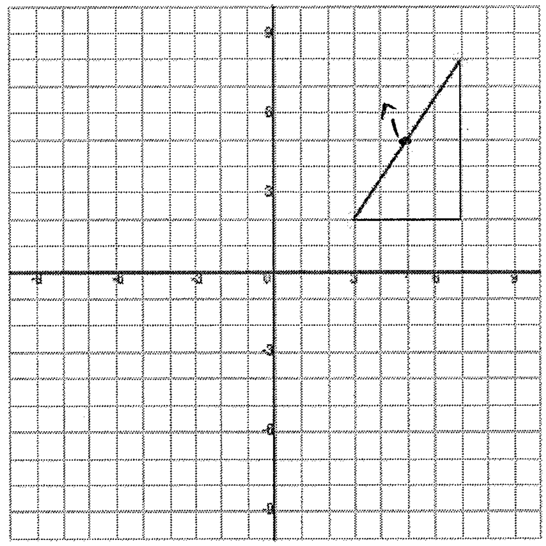
$\therefore \frac{2+8}{2} = 5$

## DIAGONAL LINE SEGMENTS

What is the midpoint of the line segment AB?

A (3, 2)

B (7, 8)



First, find the averageof $x$ values $\frac{3+7}{2} = 5$

Next, find the average of $y$ values $\frac{2+8}{2} = 5$

Midpoint = (5, 5)

Summary: To find the midpoint, find the mean of $x$-values and $y$-values.

**Formula for the Midpoint of a Line Segment:**

midpoint = (mean of $x$, mead of $y$)

midpoint = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

## Task 2: Practice

Find the Midpoint of the pair of co-ordinates given below:

a) $A(3,4),\ B(-5,2)$

$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$=\left(\frac{3+(-5)}{2},\frac{4+2}{2}\right)$

$=(-1,\ 3)$ Therefore: The midpoint is $(-1,3)$

b) $P(-7,3), Q(-9,1)$

$M=\left(\frac{-7+(-9)}{2},\ \frac{3+1}{2}\right)$

$=(-8,\ 2)$

c) $D(\frac{5}{2},-\frac{1}{3}),\ E(\frac{1}{4},\frac{7}{2})$

$M=\left(\frac{\frac{5}{2}+\frac{1}{4}}{2},\ \frac{-\frac{1}{3}+\frac{7}{2}}{2}\right)=\left(\frac{\frac{10}{4}+\frac{1}{4}}{2},\ \frac{\frac{-2}{6}+\frac{21}{6}}{2}\right)=\left(\frac{\frac{11}{4}}{2},\ \frac{\frac{19}{6}}{2}\right)$

$=\left(\frac{11}{8},\ \frac{19}{12}\right)$

## Task 3: Application

M is the midpoint of line segment UP. The coordinates of U are (-2, 3) and the coordinates of M are (1, 0). Find the coordinates of P.

$$midpoint\ M_{UP}=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$(1,0)=\left(\frac{-2+x_2}{2},\ \frac{3+y_2}{2}\right)$ $\therefore\ P(4,-3)$.

$\therefore\ \frac{-2+x_2}{2}=1$ $\frac{3+y_2}{2}=0$

$-2+x_2=2$ $x_2=4$ $3+y_2=0$

$y_2=-3$

COMPLETE: p. 66 #2, 5, 6, 12-14, 23, 29