**MCR3U1** **Date:____________________**

**Day 5: Trigonometric Identities I** **Chapter 5: Trigonometric Ratios**

A mathematical identity is an equation that is true for all values of the variables. That is to say, the Left Side is "identical" to the Right Side.

## A. The Quotient Identity

$$\tan q = \frac{\sin q}{\cos q}$$ * this follows from the unit circle

## B. The Reciprocal Identities

$$\csc q = \frac{1}{\sin q} \qquad \sec q = \frac{1}{\cos q} \qquad \cot q = \frac{1}{\tan q}$$

## C. The Pythagorean Identities

$$\sin^2 q + \cos^2 q = 1$$ * this follows from the unit circle

$$\cos^2 q = 1 - \sin^2 q$$ * important variations

$$\sin^2 q = 1 - \cos^2 q$$

**Also:**

$$\tan^2 q + 1 = \sec^2 q$$ because $$\frac{\sin^2 q}{\cos^2 q} + \frac{\cos^2 q}{\cos^2 q} = \frac{1}{\cos^2 q}$$

$$1 + \cot^2 q = \csc^2 q$$ because $$\frac{\sin^2 q}{\sin^2 q} + \frac{\cos^2 q}{\sin^2 q} = \frac{1}{\sin^2 q}$$

## How to Prove Trig Identities

1. Separate the two sides into LS and RS – it is not allowed to cross sides in a proof!
2. Express all trig functions in terms of sin and cos (using reciprocals)
3. Use the Quotient and Pythagorean identities if possible
4. Simplify the complicated side, so that it increasingly resembles the simpler side in terms of functions and exponents. Some more complicated proofs may require factoring.
5. Don't give up!

**Ex1**: Prove that $1 + \cot^2\theta = \csc^2\theta$ for all principal angles $\theta$ (except for 0° and 180° for which $\cot\theta$ and $\csc\theta$ are undefined).

**Proof**:

$LS = 1 + \cot^2\theta$

$= 1 + \frac{1}{\tan^2\theta} \rightarrow \frac{\sin^2\theta}{\cos^2\theta}$

$= 1 + \frac{\cos^2\theta}{\sin^2\theta}$ LCD $= \sin^2\theta$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} \Rightarrow \frac{1}{\sin^2\theta}$

$RS = \csc^2\theta$

$= \frac{1}{\sin^2\theta}$

LS = RS

✓ QED

Therefore LS = RS, **QED** [quod erat demonstrandum meaning "which was what had to be demonstrated," the traditional indication of the end of a proof].

***Example 1:*** Prove $\sin\theta\tan\theta\cos\theta = \sin^2\theta$

$LS = \sin\theta\tan\theta\cos\theta$

$= \sin\theta \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}$

$LS = \sin^2\theta$

$RS = \sin^2\theta$

LS = RS QED ✓

***Example 2:*** Prove $(1+\tan^2\theta)\cos^2\theta = 1$

$LS = (1+\tan^2\theta)\cos^2\theta$

$= \left(1 + \frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$

$= \left(\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$ 1

$= \frac{1}{\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta} = 1$

$RS = 1$

LS = RS ✓ QED

***Example 3:*** Prove $\sin\theta\tan\theta = \sec\theta - \cos\theta$

$LS = \sin\theta\tan\theta$

$= \sin\theta \frac{\sin\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$RS = \sec\theta - \cos\theta$

$= \frac{1}{\cos\theta} - \cos\theta$ LCD $= \cos\theta$

$= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$

$= \frac{1-\cos^2\theta}{\cos\theta} \rightarrow \sin^2\theta$

$= \sin^2\theta / \cos\theta$

$\therefore$ LS = RS QED

$\tan\theta = \frac{\sin\theta}{\cos\theta}$



***Example 4:*** Prove $\cot^2\theta(1-\cos^2\theta) = \cos^2\theta$

$LS = \cot^2\theta(1-\cos^2\theta)$

$= \frac{\cos^2\theta}{\sin^2\theta}(\sin^2\theta)$

$= \cos^2\theta$

$RS = \cos^2\theta$

$\therefore LS = RS$ QED

***Example 5:*** Prove $2\sin^2\theta - 1 = \sin^2\theta - \cos^2\theta$

$LS = 2\sin^2\theta - 1$

$= 2\sin^2\theta - (\sin^2\theta + \cos^2\theta)$

$= 2\sin^2\theta - \sin^2\theta - \cos^2\theta$

$= \sin^2\theta - \cos^2\theta$

$RS = \sin^2\theta - \cos^2\theta$

$\therefore LS = RS$ QED

***Example 6:*** Prove $\cos^2\theta = \sin^2\theta + 2\cos^2\theta - 1$

$LS = \cos^2\theta$

$RS = \sin^2\theta + 2\cos^2\theta - 1$

$= 1 - \cos^2\theta + 2\cos^2\theta - 1$

$= \cos^2\theta$

$LS = RS$ QED

***Example 7:*** Prove $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

$LS = (\sin\theta + \cos\theta)^2$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= \underline{\sin^2\theta + \cos^2\theta} + 2\sin\theta\cos\theta$

$LS = 1 + 2\sin\theta\cos\theta$

$RS = 1 + 2\sin\theta\cos\theta$

$\therefore LS = RS$ QED