Day 1: 3.1 – Reciprocal of a Linear Function

When you add, subtract, or multiply two polynomial functions, the result is another polynomial function. When you divide polynomial functions, the result is a **rational function.**

A **rational function** has the form $f(x)=\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials, and $Q(x)\neq 0$

The simplest rational function is the **reciprocal of a linear function**

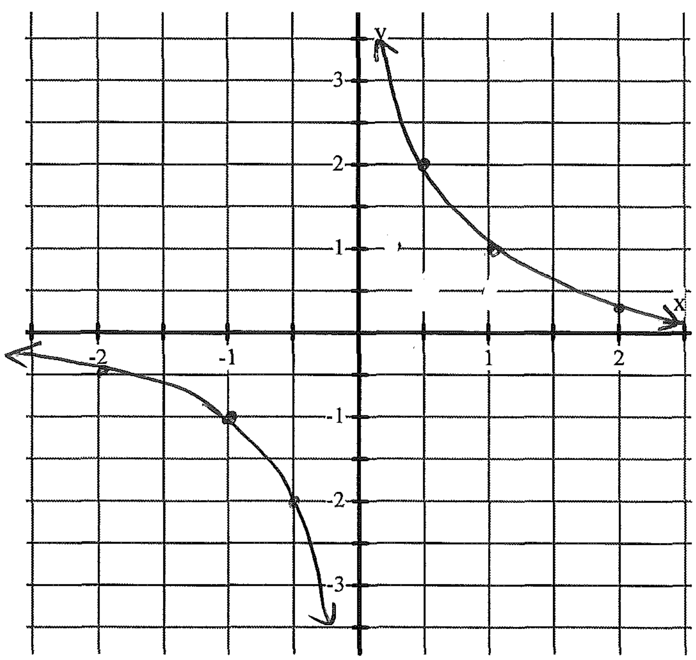
A **linear function** has the form $f(x)=mx+b$

The reciprocal of a linear function has the form $f(x)=\frac{1}{mx+b}$

Recall: Asymptote – *A line that the curve approaches (gets closer to) but never touches)*

EX 1 - The simplest form of the reciprocal of a linear function is the function $f(x)=\frac{1}{x}$. Graph the function using the table of values, then state the domain and range, and equation of the asymptotes.

X	y
-2	-0.5
-1	-1
$-\frac{1}{2}$	-2
0	undefined
$\frac{1}{2}$	2
1	1
2	0.5



Domain: $\{x \in \mathbb{R} \mid x \neq 0\}$

OR $x \in (-\infty, 0) \cup (0, \infty)$

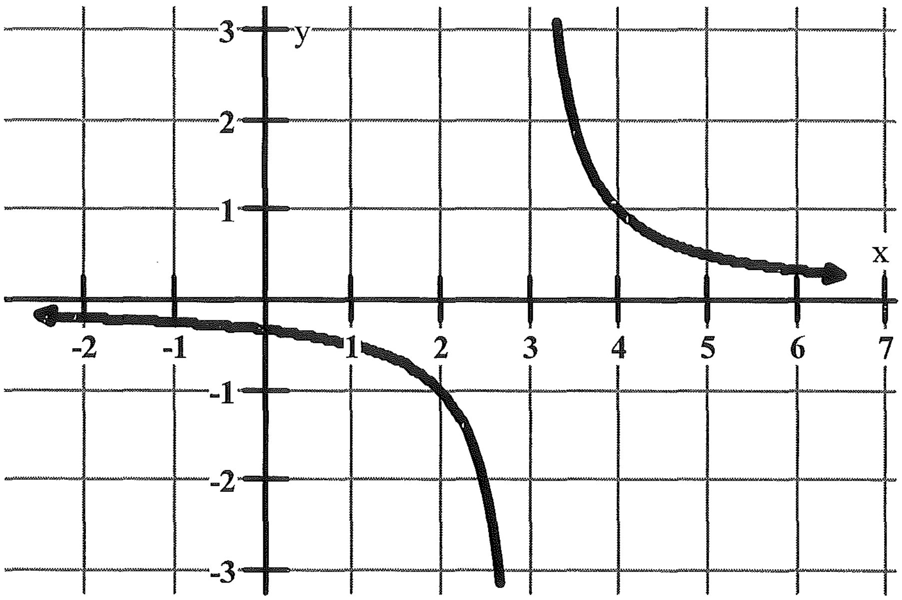
Range: $\{y \in \mathbb{R} \mid y \neq 0\}$

OR $y \in (-\infty, 0) \cup (0, \infty)$

Horizontal Asymptote: $y=0$

Vertical Asymptote: $x=0$

EX 2 – Identify the following properties for the given rational function:



Vertical Asymptote (VA)	$x=3$
Horizontal Asymptote (HA)	$y=0$
Domain	$\{x\in\mathbb{R} \mid x\neq 3\}$ OR $x\in(-\infty,3)\cup(3,\infty)$
Range	$\{y\in\mathbb{R} \mid y\neq 0\}$ OR $y\in(-\infty,0)\cup(0,\infty)$
Behaviour near VA	$x\to 3^+ \quad y\to\infty$ $x\to 3^- \quad y\to -\infty$
Behaviour near HA	$x\to\infty \quad y\to 0^+$ (from above) $x\to -\infty \quad y\to 0^-$ (from below)

To graph a reciprocal of a linear function:

a. Find the vertical asymptote (VA): *set the denominator equal to zero and solve for x*

b. Determine the horizontal asymptote (HA): *divide each term in the function by its highest power, then evaluate the function as s x→ ±∞. See what y approaches.*

c. Determine the *x & y* intercepts: *set y = 0 and solve for x, set x = 0, and solve for y*

d. Determine where the function is positive and negative: *use an interval table using the vertical asymptotes and x-intercepts for your intervals*

EX 3 – Sketch the graph of the function $f(x)=\frac{1}{2x-1}$. Then, state the domain and range.

a. VA

$2x-1=0$

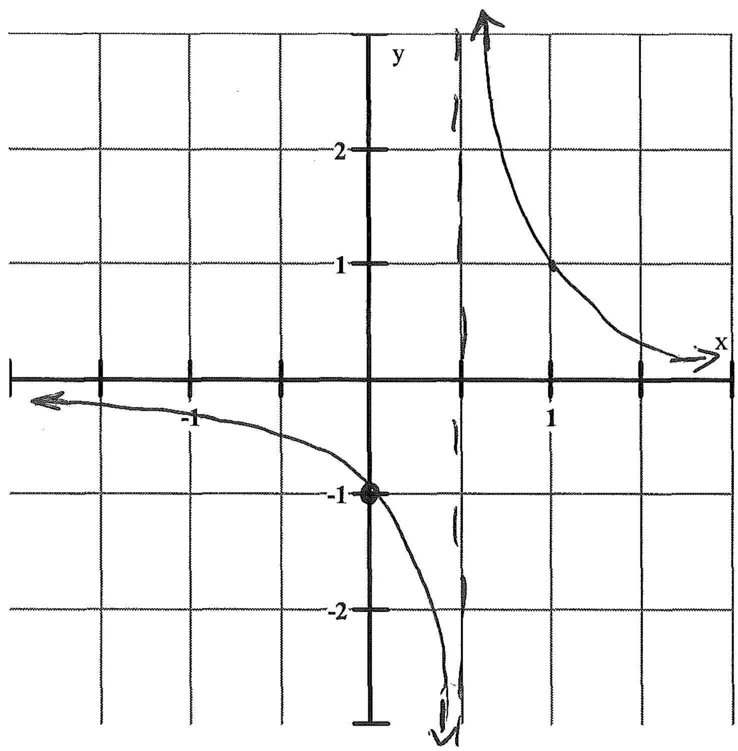
$2x=1$

$x=1/2$

b. HA

$x\to\infty \quad y\to 0^+$

$x\to-\infty \quad y\to 0^-$



c. Intercepts

x-int: set $y=0$

$0=\frac{1}{2x-1}$

$1\neq 0$

y-int: set $x=0$

$y=\frac{1}{2(0)-1}$

$=-1$

d. Interval Table

	$(-\infty, 1/2)$	$(1/2, \infty)$
$f(x)$	−	+

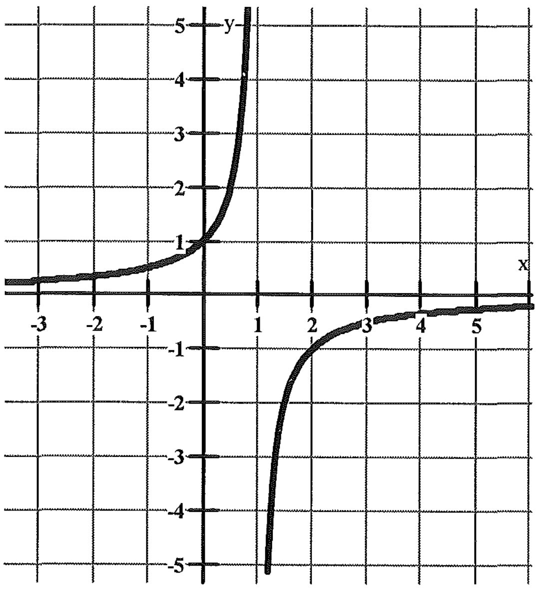
Domain: $x\in(-\infty, 1/2)\cup(1/2,\infty)$

Range: $y\in(-\infty,0)\cup(0,\infty)$

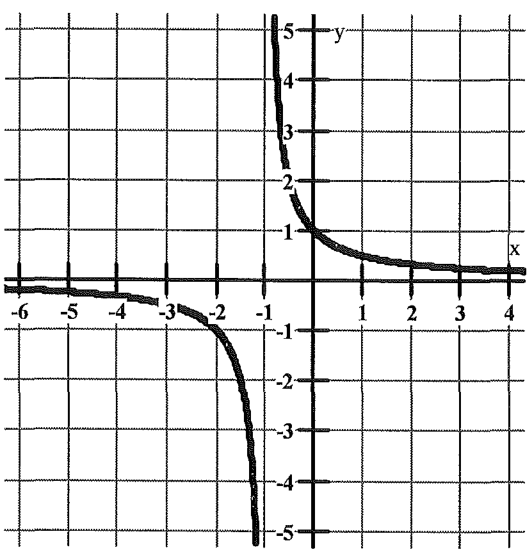
EX 4 - Increasing/Decreasing Slope

For the following functions, describe the intervals where the the <u>slope</u> is increasing and <u>slope</u> is **decreasing. Include the sign of the slope (positive or negative).**

* *The intervals are based on the <u>vertical asymptote</u>*



Interval	Change in Slope	Sign of Slope
$(-\infty, 1)$	increasing	+
$(1, \infty)$	decreasing	+



Interval	Change in Slope	Sign of Slope
$(-\infty, -1)$	decreasing	−
$(-1, \infty)$	increasing	−