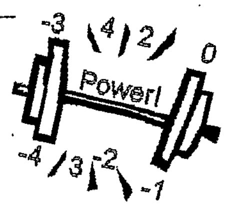
4.1 Review of Exponent Laws

Goal : to review the exponent laws

In multiplication, the terms that are multiplied together are called factors.
A repeated multiplication of equal factors can be expressed as a power



$3 \times 3 \times 3 \times 3 =$ → 3^4 is the power
→ 3 is the base
$=$ → 4 is the exponent

EXAMPLES Write each in expanded form and then evaluate to standard form

$6^3 = 6 \times 6 \times 6$
$=$

$3^2 \times 2^3 = 3 \times 3 \times 2 \times 2 \times 2$
$= 72$

$6^2 + 3^2 = 6 \times 6 + 3 \times 3$
$= 36 + 9$
$= 45$

POWER OF A NEGATIVE NUMBER

Exponents affect ONLY the number it touches in a power. Notice the difference?

$-3^2 = -9$
↳ -3×3

$(-3)^2 = (-3)(-3)$
$= 9$

EXPONENT LAWS

- **Add/Subtract Powers** → You can only add/subtract the coefficients of the **like** powers

$\boxed{a^m + a^m = 2a^m}$ $a^m + 3a^m =$

- **Multiply Powers** → To multiply powers with the **SAME base** ADD the exponents

$\boxed{a^m \times a^n = a^{m+n}}$ $x^2 \times x^3 = x^{2+3} = x^5$

- **Divide Powers** → To divide powers with the **SAME base** SUBTRACT the exponents

$\boxed{a^m \div a^n = a^{m-n}}$ $x^7 \div x^4 = x^{7-4} = x^3$

- **Power of a Power** → To simplify a power of a power MULTIPLY the exponents

$\boxed{(a^m)^n = a^{mn}}$ $(x^4)^3 = x^{4 \times 3} = x^{12}$

- **Power of a Product or Quotient** → Apply the exponent to each variable or term in the product or quotient.

$\boxed{(ab)^m = a^m b^m}$ $(xy)^3 = x^3 y^3$

$\boxed{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}}$ $\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$

Exponents Practice

1. **Simplify** each of the following in exponent form **and evaluate** to standard form.

a) $4\times4\times4\times4\times4\times4$

$= 4^6$

$= 4096$

b) $-3\times-3\times-3$

$= (-3)^3$

$= -27$

c) $(5\times5\times5)+(6\times6)$

$= 5^3 + 6^2$

$= 125+36$

$= 161$

d) $(-9\times-9)-(7\times7\times7)$

$= (-9)^2 - 7^3$

$= 81 - 343$

$= -262$

e) $(-2\times-2\times-2\times-2)\times(5\times5)$

$= (-2)^4 \times 5^2$

$= 16 \times 25$

$= 400$

f) $\left(\frac{3}{5}\right)\times\left(\frac{3}{5}\right)$ **leave as a fraction**

$= \left(\frac{3}{5}\right)^2 = \frac{9}{25}$

2. Write each of the following in **expanded** form and evaluate to **standard** form.

a) 3^4

$= 3\times3\times3\times3$

$= 81$

b) $(-5)^3$

$= (-5)(-5)(-5)$

$= -125$

c) $\left(\frac{-2}{3}\right)^3 = \left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right)$

$= \frac{-8}{27}$

d) $(-4)^2+3^2$

$= (-4)(-4)+3\times3$

$= 16+9$

$= 25$

e) 2^5-4^2

$= 2\times2\times2\times2\times2-4\times4$

$= 32-16$

$= 16$

f) $(-2)^4\div8$

$= (-2)(-2)(-2)(-2)\div8$

$= \frac{16}{8} = 2$

3. **Simplify** each expression.

a) $x^4\times x^2$

$= x^6$

b) $(2xy)^2 = 2^2x^2y^2$

$= 4x^2y^2$

c) $(2xy)(xy) = 2x^2y^2$

d) $y^6\times y^3\div y^7$

$= \frac{y^9}{y^7} = y^2$

e) $x^3(x^2)-x^2(x)$

$= x^5 - x^3$

f) $(-2y^3)^3$

$= (-2)^3y^9 = -8y^9$

g) $(3x^2)^3$

$= 3^3x^6 = 27x^6$

h) $\frac{x^8}{x^3} = x^5$

i) $\left(\frac{x^5}{x^2}\right)^2 = \frac{x^{10}}{x^4}$

j) $\left(\frac{x^2}{y^2}\right)+2\left(\frac{x}{y}\right)^2$

$= \frac{x^2}{y^2} + 2\frac{x^2}{y^2} = 3\frac{x^2}{y^2}$

k) $\left(\frac{m^5n^2}{m^3}\right)^2 = (m^2n^2)^2$

$= m^4n^4$

l) $\frac{-6a^7b^4}{3a^2b^2}$

$= -2a^5b^2$

Why Was the Engineer Driving the Train Backwards?

Find the missing factor in each exercise below. Find your answer in the set of answers to the right of that exercise. Write the letter next to your answer in the box containing the number of that exercise.

1. $x^8 = (x^5)(\underline{x^3})$ H
2. $24x^5 = (6x^2)(\underline{4x^3})$ O
3. $-12x^4 = (3x^3)(\underline{-4x})$ I
4. $20x^7 = (-4x^2)(\underline{-5x^5})$ A

(T) $4x^5$	(N) x^6
(A) $-5x^5$	(O) $4x^3$
(H) x^3	(R) $-4x^8$
(E) $-5x^3$	(I) $-4x$

5. $a^5b^8 = (a^2b^3)(\underline{a^3b^5})$ E
6. $4a^2b^6 = (2ab^2)(\underline{2ab^4})$ O
7. $-15a^7b^4 = (-3a^4b)(\underline{5a^3b^3})$ K
8. $72a^{10}b^3 = (-6a^5b^2)(\underline{-12a^5b})$ H

(P) a^2b^2	(E) a^3b^5
(V) $5a^3b^3$	(A) $-12a^2b^4$
(L) $2ab^7$	(H) $-12a^5b$
(O) $2ab^4$	(K) $5a^5b^3$

9. $x^5y^3 = (x^2)(\underline{x^3y^3})$ A
10. $-6x^2y^7 = (-2y)(\underline{3x^2y^6})$ O
11. $14x^9y^6 = (-7x^2y^6)(\underline{-2x^7})$ L
12. $27x^4y^3 = (9x^4y)(\underline{3y^2})$ E

(V) $-3y^4$	(O) $3x^2y^6$
(L) $-2x^7$	(T) $3x^2y^3$
(S) $-2x^6y$	(A) x^3y^3
(B) x^2y^4	(E) $3y^2$

13. $-3u^4v^2 = (u^2v)(\underline{-3u^2v})$ T
14. $32uv^5 = (-16v^2)(\underline{-2uv^3})$ D
15. $121u^2v^3 = (11u^2v)(\underline{11v^2})$ M
16. $-6u^3v^{12} = (2uv)(\underline{-3u^2v^{11}})$ C

(R) $-2uv^6$	(R) $-3u^2v^4$
(M) $11v^2$	(C) $-3u^2v^{11}$
(P) $11uv^3$	(E) $3u^2v^6$
(T) $-3u^2v$	(D) $-2uv^3$

8	12	1	9	14	4	11	2	16	6	15	10	13	3	7	5
H	E	H	A	D	A	L	O	M	O	C	O	T	I	K	E

OBJECTIVE 3–c: To find a missing factor of a monomial.

4.2 Zero and Negative Exponents

Goal: to determine the meaning of zero and negative exponents

Complete the following chart. Evaluate each to standard form. Leave as whole numbers or fractions.

Expression to be Simplified	Write in Expanded Form	Using Exponent Laws
$\frac{2^3}{2^1}$	$=\frac{2\times2\times2}{2}$ $=4$	$2^{3-1}=2^2$ $=4$
$\frac{2^3}{2^2}$	$=\frac{2\times2\times2}{2\times2}=2$	$2^{3-2}=2^1=2$
$\frac{2^3}{2^3}$	$=\frac{2\times2\times2}{2\times2\times2}=1$	$2^{3-3}=2^0=1$
$\frac{2^3}{2^4}$	$=\frac{2\times2\times2}{2\times2\times2\times2}=\frac{1}{2}$	$2^{3-4}=2^{-1}=\frac{1}{2}$
$\frac{2^3}{2^5}$	$=\frac{2\times2\times2}{2\times2\times2\times2\times2}=\frac{1}{4}$	$2^{3-5}=2^{-2}=\frac{1}{4}$

HOW is the exponent law expression **RELATED TO** the expanded form expression?

$a^{-x}=\frac{1}{a^x}$ Since $2^{-1}=\frac{1}{2^1}$ and $2^{-2}=\frac{1}{2^2}$

What do you notice about the result of an expression with an **exponent of zero?**

equals 1

What do you notice about the result of an expression with an **exponent that is negative?**

If exponent is negative, the evaluated answer is $\frac{1}{\square}$.

THE ZERO EXPONENT

Any number (or expression) divided by itself is equal to 1

Use exponent laws to evaluate each of the following:

a) $\frac{2^3}{2^3}=2^{3-3}=2^0=1$ b) $\frac{3^2}{3^2}=3^{2-2}=3^0=1$ c) $\frac{x^4}{x^4}=1$

Therefore, for zero exponents: Any **BASE** raised to an **exponent of zero** is equal to 1

$$\boxed{a^0=1}$$

EXAMPLES - Evaluate.

$1^0 =$ 1 $x^0 =$ 1 $3\times 2^0 =$ 3 $x^0 y =$ y

THE NEGATIVE EXPONENT

Any BASE raised to a NEGATIVE exponent is equal to the reciprocal of the base raised to the same positive exponent.

$$\boxed{a^{-m} = \frac{1}{a^m}} \quad \text{and} \quad \boxed{\frac{1}{a^{-m}} = a^m}$$

Use exponent laws to simplify each of the following. Then evaluate to standard form.

a) $\frac{2^3}{2^4} = 2^{-1} = \frac{1}{2^1}$

b) $\frac{3^2}{3^5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

c) $\frac{4^5}{4^7} = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

EXAMPLES
Simplify and evaluate.

$7^{-1} = \frac{1}{7}$ $(-8)^{-2} = \frac{1}{64}$ $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27}$

EXERCISE: Complete the following table.

Exponent Form	3^2	5^{-1}	10^0	3^{-3}	3^{-1}	2^{-n}	5^{-m}	$(-1225)^0$
Simplified Form	9	$\frac{1}{5}$	1	$\frac{1}{27}$	$\frac{1}{3}$	$\frac{1}{2^n}$	$\frac{1}{5^m}$	1

SIMPLIFYING EXPRESSIONS

The rules for positive exponents also work for zero and negative exponents. Continue to follow the rules for order of operations (BEDMAS) when simplifying & evaluating.

EXAMPLES
Simplify and evaluate each of the following:

$3^3 \times 3^{-5} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$\frac{(-2)^2}{(-2)^{-3}} = (-2)^{2-(-3)} = (-2)^5 = \cancel{-32}\ 32$

$\left(\frac{3^2}{3^4}\right)^2 = \frac{3^4}{3^8} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

Zero & Negative Exponents Practice

1. Complete the following table. Express your answers as whole numbers or fractions.

Exponent Form	5^2	5^{-2}	10^3	10^{-3}	x^4	x^{-4}	2^x	2^{-x}
Simplified Form	25	$\frac{1}{5^2} = \frac{1}{25}$	1000	$\frac{1}{1000}$	x^4	$\frac{1}{x^4}$	2^x	$\frac{1}{2^x}$

2. Evaluate. Express your answers as whole numbers or fractions.

a) $12^0 = 1$

b) $8^{-1} = \frac{1}{8}$

c) $(-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{16}$

d) $(-12)^0 = 1$

e) $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = \frac{16}{1} = 16$

f) $\left(\frac{1}{3}\right)^{-3} = \left(\frac{3}{1}\right)^3 = 27$

g) $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27}$

h) $\frac{1}{2^{-1}} = 2^1 = 2$

3. Evaluate. Rewrite negative exponents and evaluate as fractions.

a) $-(16)^0 = -1$

b) $4^{-4} = \frac{1}{4^4} = \frac{1}{64}$

c) $(-3)^3 = -27$

d) $(-11)^1 = -11$

e) $-(-6)^3 = -(-216) = 216$

f) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

g) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

h) $3^0 = 1$

4. Simplify each as a single power, then evaluate. Express your answers as fractions.
The first two have been done for you.

d) $8^3 \times 8 = 8^{3+1}$
$= 8^4$
$= 4096$

e) $\frac{1}{(2^4)^3} = \frac{1}{2^{4\times3}}$
$= \frac{1}{2^{12}}$
$= 2^{-12}$
$= \frac{1}{4096}$

f) $(10^{-2})^2 = 10^{-4} = \frac{1}{10^4} = \frac{1}{10000}$

g) $6^2 \div 6^5 = 6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

h) $\left(\frac{1}{2^4}\right)\left(\frac{1}{2^2}\right) = \frac{1}{2^{4+2}} = \frac{1}{2^6} = \frac{1}{64}$

i) $\left(\frac{1}{5}\right)^{-9} \times \left(\frac{1}{5}\right)^7 = \left(\frac{1}{5}\right)^{-2} = \left(\frac{5}{1}\right)^2 = 25$

Hwk: pg 199 # 1, 2, 3, 6, 7, 8 Thinking 19.