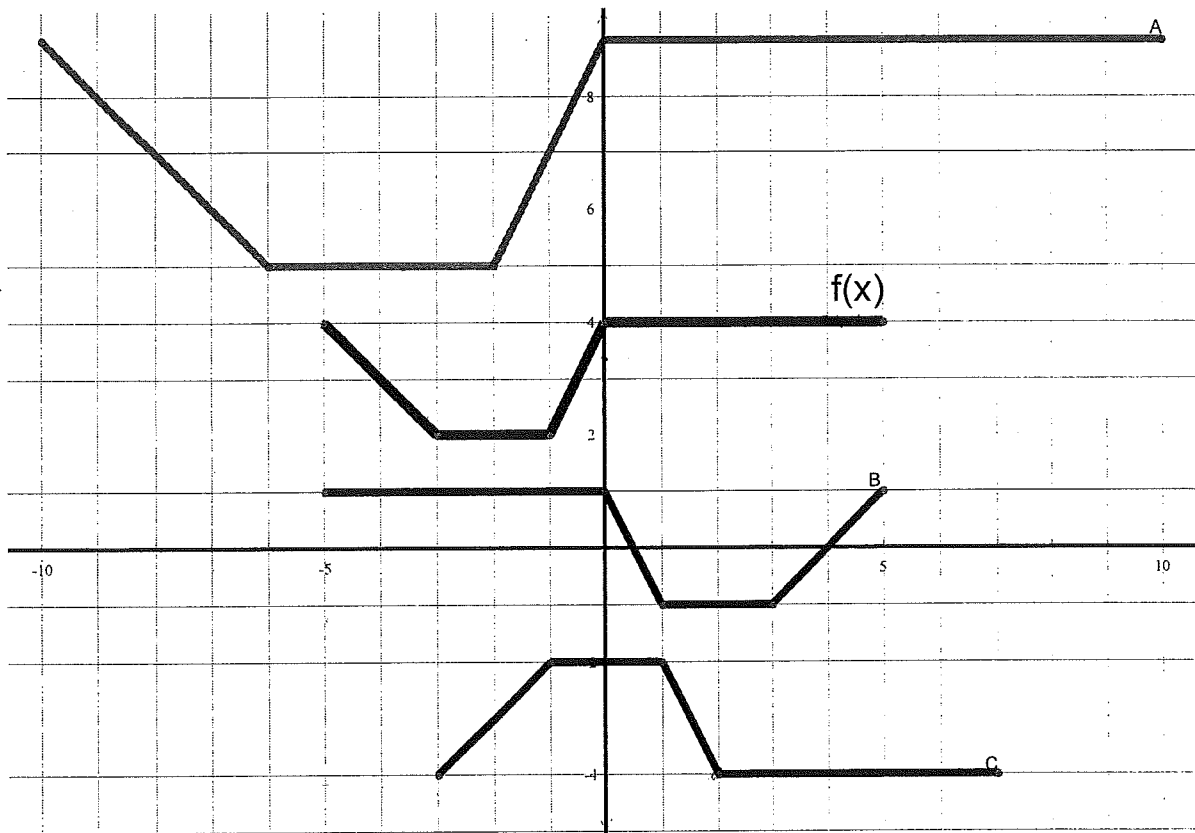


FUNCTION TRANSFORMATIONS

1. The graph of $f(x)$ is given. Match each graph with one of the equations provided.



- i) $f(x-2)+3$
- ii) $-f(x-2)-1$
- iii) $-f(x-2)$
- iv) $\frac{1}{2}f(2x)+1$
- v) $2f\left(\frac{1}{2}x\right)+1$
- vi) $-f(x-2)-3$
- vii) $f(x-2)-1$
- viii) $2f(2x)+1$
- ix) $f(-x)-3$
- x) $\frac{1}{2}f\left(\frac{1}{2}x\right)+1$

A: $2f\left(\frac{1}{2}x\right)+1$
 ↓
 $(0, 4) \rightarrow (0, 9)$
 $(5, 4) \rightarrow (10, 9)$
 ↓
 x multiplied by $\frac{1}{2}$
 y multiplied by 2
 and translated
 1 up.

B: $f(-x)-3$
 reflected
 about
 y -axis
 then translated
 down.

C: $-f(x-2)$
 reflected
 about x -axis
 $(0, 4) \rightarrow (2, -4)$
 translated 2 right

TRANSFORMATION MAPPING NOTATION: TRANSLATIONS

1. Express each in transformation mapping notation. i.e. $T: (x, y) \rightarrow (x+p, y+q)$

- a) $y = f(x) + 5$ b) $y = f(x-4)$ c) $y = f(x+7)$ d) $y = f(x) - 1$
 e) $y = f(x+2) + 13$ f) $y = f(x-8) + 2$ g) $y = f(x+10) - 6$ h) $y = f(x-19) - 3$
 i) $y = f(x+12) + 18$ j) $y = f(x-9) - 25$ k) $y = x^2 + 8$ l) $y = (x-11)^3$
 m) $y = |x-2|$ n) $y = \sqrt{x+8} - 12$ o) $f(x) = 2^{x+5} + 1$ p) $g(x) = \frac{1}{x+17}$
 q) $h(x) = \sqrt{9-(x-5)^2} - 45$ r) $f(x) = \sqrt{x} + 23$ s) $y = 2^{x+9}$ t) $y = \frac{1}{x-1} + 3$

2. Express each of the transformations above in words, using the correct language.

3. Express each of the transformations in words, using the correct language.

- a) $T: (x, y) \rightarrow (x+1, y-8)$ b) $T: (x, y) \rightarrow (x-3, y+7)$

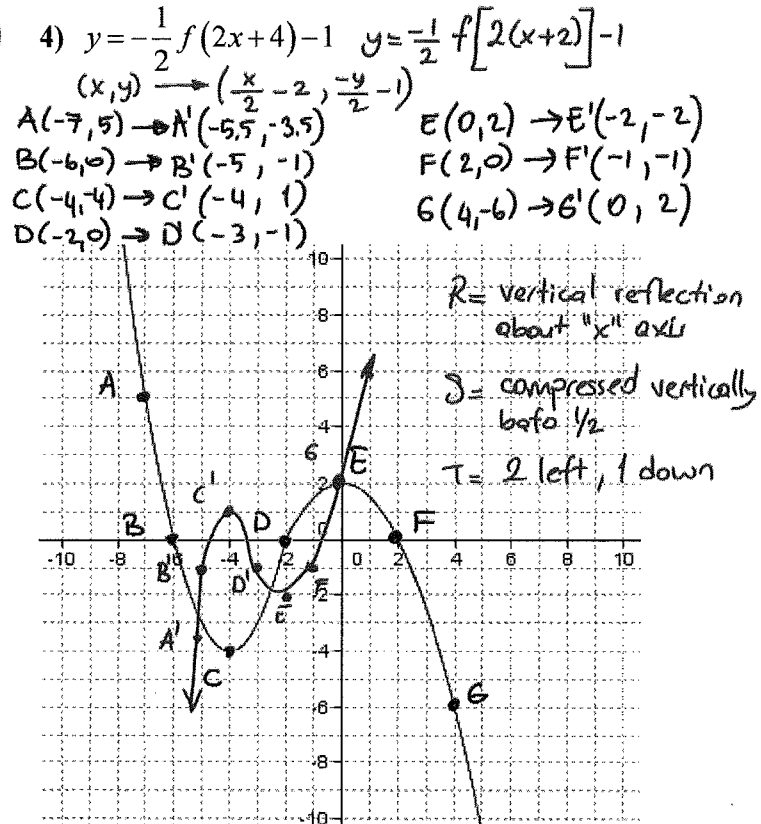
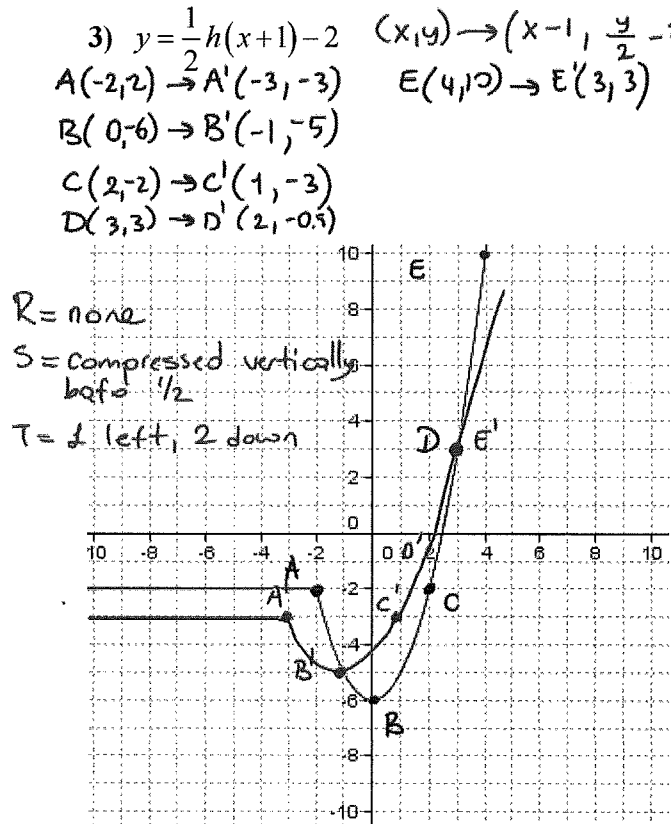
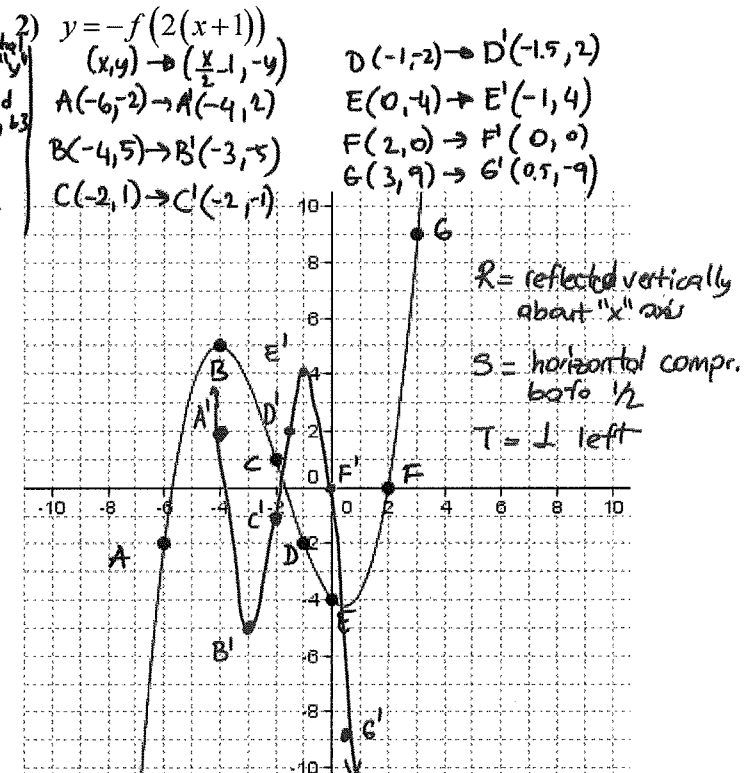
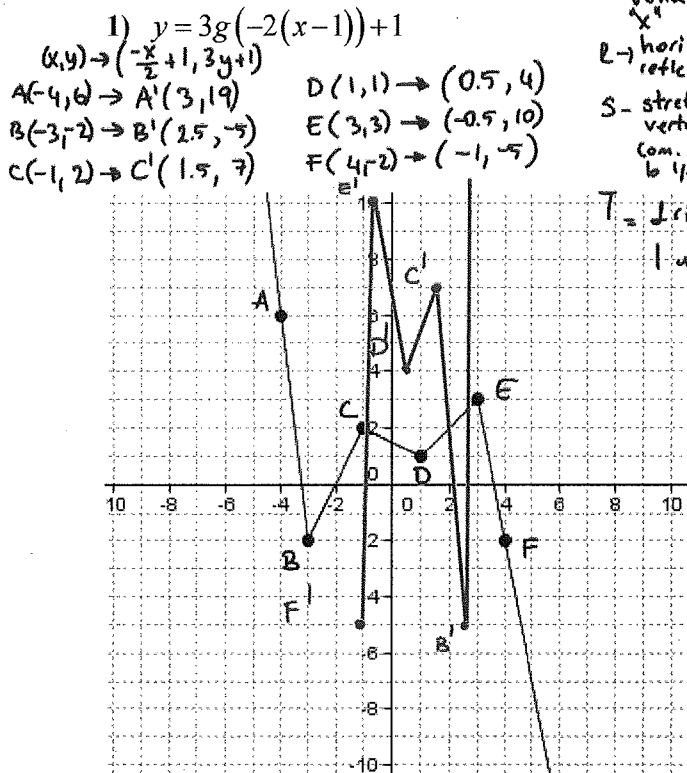
ANSWERS:

- 1.a) $T: (x, y) \rightarrow (x, y+5)$ b) $T: (x, y) \rightarrow (x+4, y)$ c) $T: (x, y) \rightarrow (x-7, y)$
 d) $T: (x, y) \rightarrow (x, y-1)$ e) $T: (x, y) \rightarrow (x-2, y+13)$ f) $T: (x, y) \rightarrow (x+8, y+2)$
 g) $T: (x, y) \rightarrow (x-10, y-6)$ h) $T: (x, y) \rightarrow (x+19, y-3)$ i) $T: (x, y) \rightarrow (x-12, y+18)$
 j) $T: (x, y) \rightarrow (x+9, y-25)$ k) $T: (x, y) \rightarrow (x, y+8)$ l) $T: (x, y) \rightarrow (x+11, y)$
 m) $T: (x, y) \rightarrow (x+2, y)$ n) $T: (x, y) \rightarrow (x-8, y-12)$ o) $T: (x, y) \rightarrow (x-5, y+1)$
 p) $T: (x, y) \rightarrow (x-17, y)$ q) $T: (x, y) \rightarrow (x+5, y-45)$ r) $T: (x, y) \rightarrow (x, y+23)$
 s) $T: (x, y) \rightarrow (x-9, y)$ t) $T: (x, y) \rightarrow (x+1, y+3)$ 2.a) vertical translation up 5 units
 b) horizontal translation right 4 units c) horizontal translation left 7 units
 d) vertical translation down 1 unit
 e) horizontal translation left 2 units and vertical translation up 13 units
 f) horizontal translation right 8 units and vertical translation up 2 units
 g) horizontal translation left 10 units and vertical translation down 6 units
 h) horizontal translation right 19 units and vertical translation down 3 units
 i) horizontal translation left 12 units and vertical translation up 18 units
 j) horizontal translation right 9 units and vertical translation down 25 units
 k) vertical translation up 8 units l) horizontal translation right 11 units
 m) horizontal translation right 2 units
 n) horizontal translation left 8 units and vertical translation down 12 units
 o) horizontal translation left 5 units and vertical translation up 1 unit
 p) horizontal translation left 17 units
 q) horizontal translation right 5 units and vertical translation down 45 units
 r) vertical translation up 23 units s) horizontal translation left 9 units
 t) horizontal translation right 1 unit and vertical translation up 3 units
 3.a) horizontal translation right 1 unit and vertical translation down 8 units
 b) horizontal translation left 3 units and vertical translation up 7 units

Practice Transformations Given a Graph

List the transformations.

Apply the transformations to key points on the graph.



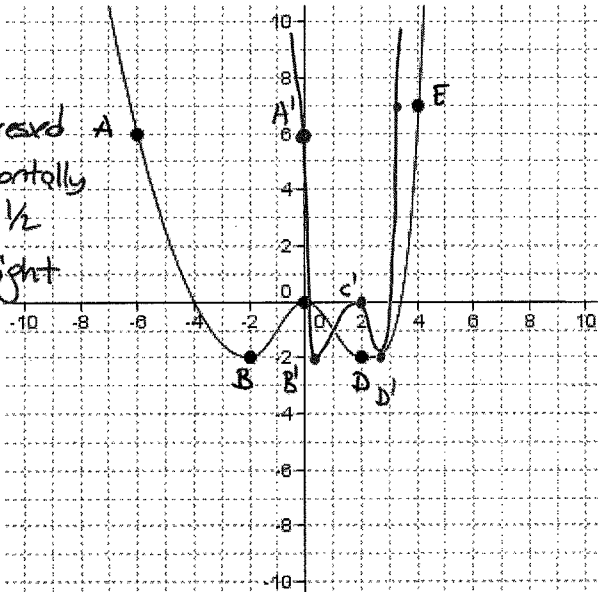
5) $y = f(3x-6)$ $y = f[3(x-2)]$
 $(x,y) \rightarrow (\frac{x}{3}+2, y)$

$A(-6,6) \rightarrow A'(0,6)$ $D(2,-2) \rightarrow D'(\frac{8}{3}, -2)$
 $B(-2,2) \rightarrow B'(\frac{4}{3}, 2)$ $E(4,7) \rightarrow E'(\frac{10}{3}, 7)$
 $C(0,0) \rightarrow C'(2,0)$

R: none

S: compressed horizontally
before $\frac{1}{2}$

T: 2 right



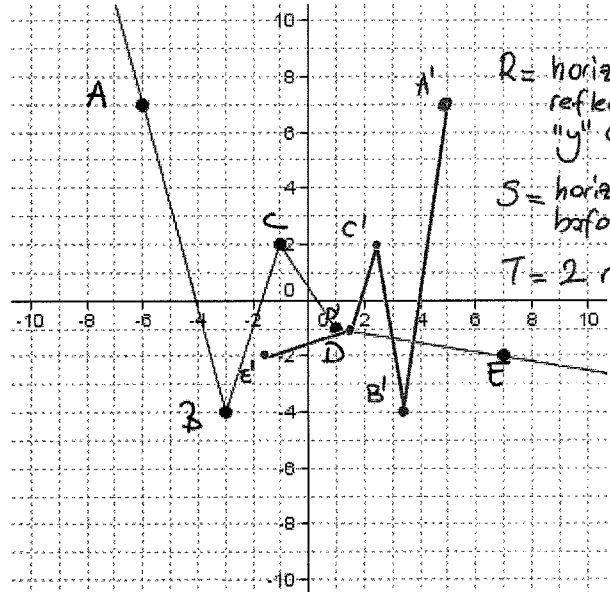
6) $y = f(-2x+4)$ $y = f[-2(x-2)]$
 $(x,y) \rightarrow (-\frac{x}{2}+2, y)$

$A(-6,7) \rightarrow A'(5,7)$ $D(1,-1) \rightarrow D'(1.5,-1)$
 $B(-3,-4) \rightarrow B'(3.5,-4)$ $E(7,-2) \rightarrow E'(-1.5,-2)$
 $C(-1,2) \rightarrow C'(2.5,2)$

R = horizontal reflection about "y" axis

S = horizontal comp before $\frac{1}{2}$

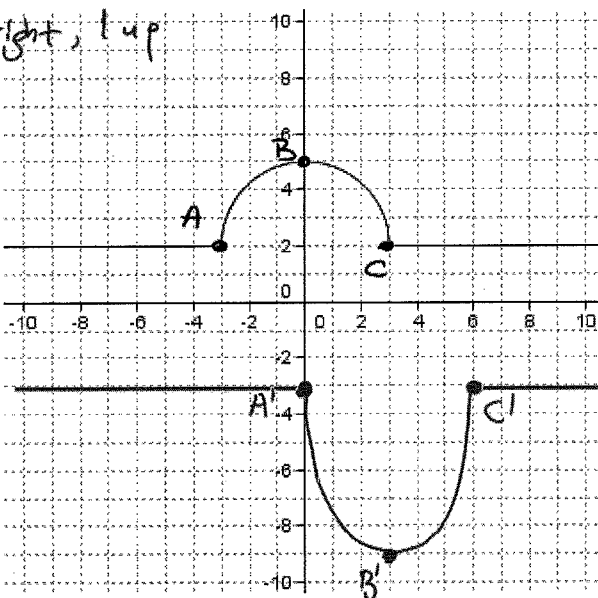
T = 2 right



7) $y = -2f(x-3)+1$ $(x,y) \rightarrow (x+3, -2y+1)$

R = vertical reflection about "x" axis
 S = vertical stretch before 2
 T = 3 right, 1 up

$A(-3,2) \rightarrow A'(0,-3)$
 $B(0,5) \rightarrow B'(3,-9)$
 $C(3,2) \rightarrow C'(6,-3)$



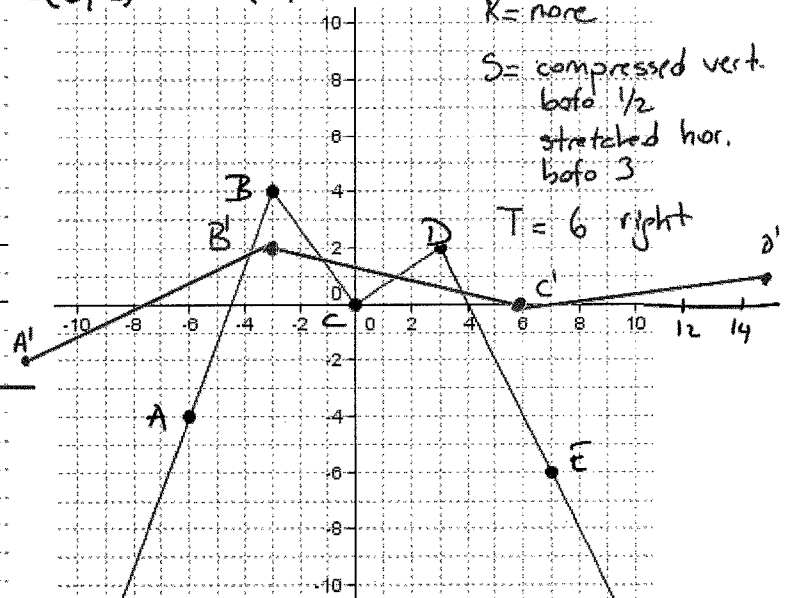
8) $y = \frac{1}{2}f(\frac{1}{3}x-2)$ $y = \frac{1}{2}f[\frac{1}{3}(x-6)]$

$(x,y) \rightarrow (3x+6, \frac{y}{2})$
 $A(-6,-4) \rightarrow A'(-12,-2)$ $D(3,2) \rightarrow D'(15,1)$
 $B(-3,4) \rightarrow B'(-3,2)$ $E(7,-4) \rightarrow E'(27,-3)$
 $C(0,0) \rightarrow C'(6,0)$

R = none

S = compressed vert before $\frac{1}{2}$
 stretched hor. before 3

T = 6 right



Transformations Practice

$$y = af[k(x-d)] + c$$

Transformation(s) given in the function notation	Transformation parameter values (a, k, d, c)	What is(are) the transformation(s)?	Parent function	The result of applying the transformation(s) to the parent function
$y = -7f(2(x+1))$	$a = -7$ $k = 2$ $d = -1$	<ul style="list-style-type: none"> reflect about x-axis vertically stretch by a factor of 7 horizontally comp. by a factor of $\frac{1}{2}$ translate 1 left 	$f(x) = \frac{1}{x}$	$y = -\frac{7}{2(x+1)}$
$y = f(x-1) - 1$	$d = 1$ $c = -1$	translate 1 unit right and 1 unit down.	$f(x) = \sqrt{x}$	$y = \sqrt{x-1} - 1$
$y = f(3(x-2))$	$k = 3$ $d = 2$	<ul style="list-style-type: none"> horizontally comp. by a factor of $\frac{1}{3}$ translate 2 right 	$f(x) = \sqrt{x}$	$y = \sqrt{3x-6} = \sqrt{3(x-2)}$
$y = f(x+1) + 3$	$d = -1$ $c = 3$	Translate left 1 unit and up 3 units	$f(x) = \sqrt{x}$	$y = \sqrt{x+1} + 3$
$y = 2f(-2x)$	$a = 2$ $k = -2$	<ul style="list-style-type: none"> reflect about y-axis vertical stretch by a factor of 2 horizontal comp. by a factor of $\frac{1}{2}$ 	$f(x) = \sqrt{x}$	$y = 2\sqrt{-2x}$
$y = \frac{1}{2}f(x+3)$	$a = \frac{1}{2}$ $d = -3$	<ul style="list-style-type: none"> reflect about x-axis vert. comp. by a factor of $\frac{1}{2}$ translate 3 left 	$f(x) = \sqrt{x}$	$y = -\frac{1}{2}\sqrt{x+3}$
$y = 3f\left[\frac{1}{2}(x-1)\right]$	$a = 3$ $k = \frac{1}{2}$ $d = 1$	<ul style="list-style-type: none"> vertically stretched by a factor of 3 horizontally stretched by a factor of 2 translate 1 right 	$f(x) = \sqrt{x}$	$y = 3\sqrt{\frac{1}{2}(x-1)}$
$y = 2f(3x) - 4$	$a = 2$ $k = 3$ $c = -4$	<ul style="list-style-type: none"> vertical stretch by a factor of 2 horizontal comp. by a factor of $\frac{1}{3}$ translate 4 down 	$f(x) = x $	$y = 2\sqrt{3x-4}$
$y = 2f(x+6) - 1$	$a = 2$ $d = -6$ $c = -1$	<ul style="list-style-type: none"> vertically stretched by a factor of 2 translate 6 units left, 1 down 	$f(x) = x $	$y = 2 x+6 - 1$

by a factor of

Transformations Practice

$$y = af[k(x-d)] + c$$

Transformation(s) given in the function notation	Transformation parameter values (a, k, d, c)	What is (are) the transformation(s)?	Parent function	The result of applying the transformation(s) to the parent function
$y = -2f(x)$	$a = -2$	Reflection in the x-axis, vertical stretch by factor of 2	$f(x) = x^2$	$y = -2(x)^2$
$y = f\left(-\frac{1}{3}x\right)$	$k = -\frac{1}{3}$	Reflection about y-axis horizontally stretched by factor of 3.	$f(x) = x^2$	$y = \left(-\frac{1}{3}x\right)^2$
$y = 2f(x) - 5$	$a = 2, c = -5$	vertically stretched by a factor of 2 translate 5 down.	$f(x) = x^2$	$y = 2x^2 - 5$
$y = f(x-3) - 2$	$d = 3, c = -2$	Translate right 3 units and down 2 units	$f(x) = x^2$	$y = (x-3)^2 - 2$
$y = -\frac{1}{2}f(x+4) - 5$	$a = -\frac{1}{2}, d = -4, c = -5$	Reflection about x-axis, vertically compressed by a factor of 1/2, translate 4 left, 5 down	$f(x) = x^2$	$y = -\frac{1}{2}(x+4)^2 - 5$
$y = -5f(x)$	$a = -5$	Reflection about y-axis vertically stretched by factor of 5.	$f(x) = \frac{1}{x}$	$y = \frac{-5}{x}$
$y = f(x) - 1$	$c = -1$	translate 1 unit down.	$f(x) = \frac{1}{x}$	$y = \frac{1}{x} - 1$
$y = f(x-d) + 3$	$d = 2, c = 3$	translate 2 units right 3 units up.	$f(x) = \frac{1}{x}$	$y = \frac{1}{x-2} + 3$
$y = -f(2x) - 5$	$a = -1, k = 2, c = -5$	Reflection in the x-axis, horizontal compression by factor of 1/2, and translate down 5 units	$f(x) = \frac{1}{x}$	$y = \frac{-1}{2(x)} - 5$