

Transformations

Part 7 - Transformations of $y = af[k(x-d)]+c$

Transformations can be combined so that more than one transformation is performed on a given function. Apply the transformations in the order RST:

- 1) Reflect: reflect about the x and/or y axis
- 2) Stretch: stretch vertically and/or horizontally
- 3) Translate: translate left or right and/or up or down

Recall the following transformation rules:

- a: vertical stretch by a factor of a (reflect about the x-axis if a is negative)
 k: horizontal stretch by a factor of $1/k$ (reflect about the y-axis if k is negative)
 d: translate right d units (left if d is negative)
 c: translate up c units (down if c is negative)

- a) What transformations have been performed on $y = x$ to give $y = 4(x-6)$?

vertical stretch by a factor of 4, translate right 6 units.

- b) What transformations have been performed on $y = x^2$ to give $y = (-x)^2 + 5$?

reflect about y-axis, translate 5 units up.

- c) What transformations have been performed on $y = \sqrt{x}$ to give $y = -\sqrt{\frac{1}{6}x}$?

Reflected about x-axis, horizontally stretched by a factor of 6.

- d) What transformations have been performed on $y = \sqrt{x}$ to give $y = 3\sqrt{2x-2} - 4$?

Hint: factor k first! $3\sqrt{2(x-1)} - 4$

vertically stretched by a factor of 3. Horizontally compressed by a factor of $\frac{1}{2}$, translate 1 unit right and 4 down.

- e) What transformations have been performed on $y = x^2$ to give $y = -2\left(-\frac{1}{3}x-3\right)^2 + 7$? Hint:

factor k first! $y = -2\left(-\frac{1}{3}(x+9)\right)^2 + 7$

reflect about x and y-axis. vertically stretched by a factor of 2 and horizontally stretched by a factor of 3. translate 9 left and 7 up.

Transformations

Part 8 - The Graph of $y = af[k(x-d)]+c$

First determine the parent function $y = f(x)$. This gives you the general shape of the graph.

Determine the values of a , k , d , and c .

Note: use the actual value of k , not $1/k$

Determine the key points (and asymptotes, if they exist) for the parent function.

Apply the mapping formula $(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$ to each ordered pair (x, y)

For the reciprocal function, $y = 1/x$, there is a vertical asymptote at $x = d$ and a horizontal asymptote at $y = c$.

Graph using a smooth curve and arrows where appropriate. Don't forget to label the axes and scales.

Ex. 1. Graph $y = -3\sqrt{2(x+1)} + 4$

$$(x, y) \rightarrow \left(\frac{x}{2} - 1, -\frac{1}{3}y + 4\right)$$

Parent function: $y = \sqrt{x}$

$a = -3, k = 2, d = -1, c = 4$

Key points and mapping:

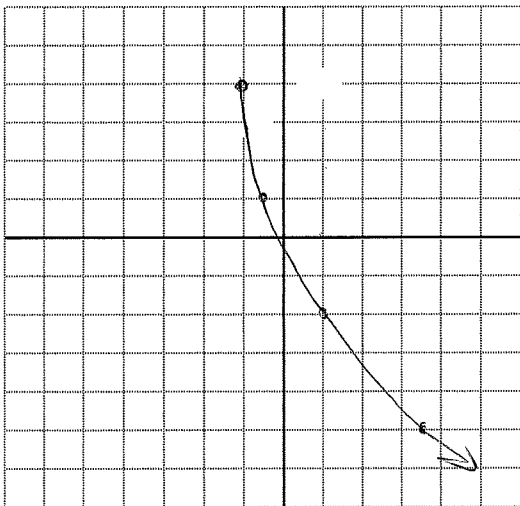
$$(0, 0) \rightarrow (0/2 + -1, -3(0) + 4) \rightarrow (-1, 4)$$

$$(1, 1) \rightarrow (1/2 + -1, -3(1) + 4) \rightarrow (-0.5, 1)$$

$$(4, 2) \rightarrow (4/2 + -1, -3(2) + 4) \rightarrow (1, -2)$$

$$(9, 3) \rightarrow (9/2 + -1, -3(3) + 4) \rightarrow (3.5, -5)$$

Asymptotes: none



Ex 2. Graph $y = \frac{2}{x-3}$

Parent function: $y = \frac{1}{x}$

$a = 2, k = 1, d = 3, c = 0$

Key points and mapping:

$$(1, 1) \rightarrow (1/1 + 3, 2(1) + 0) \rightarrow (4, 2)$$

$$(-1, -1) \rightarrow (-1/1 + 3, 2(-1) + 0) \rightarrow (2, -2)$$

Asymptotes: VA: $x = 3$

HA: $y = 0$

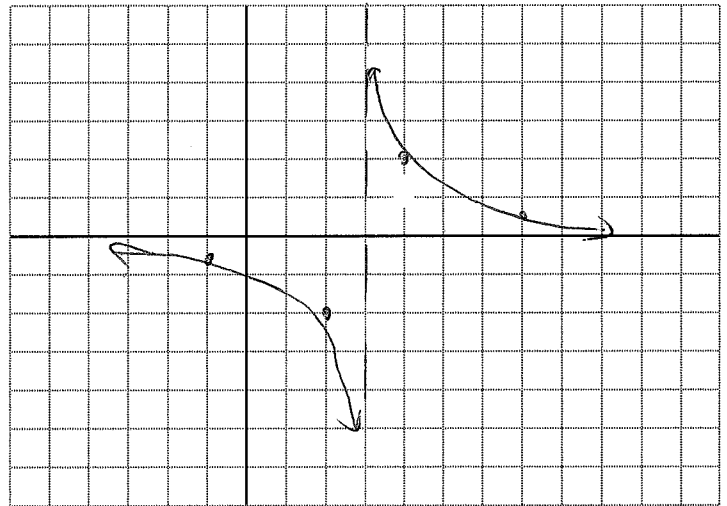
Handwritten notes for Ex 2:

$$(4, 0.25) \rightarrow$$

$$(7, 0.5) \rightarrow$$

$$(-4, -0.5) \rightarrow$$

$$\rightarrow (-1, -0.5) \rightarrow$$



Practice:

a) Graph $y = [2(x+1)]^2 - 4$

$(x,y) \rightarrow (\frac{x}{2}-1, y-4)$

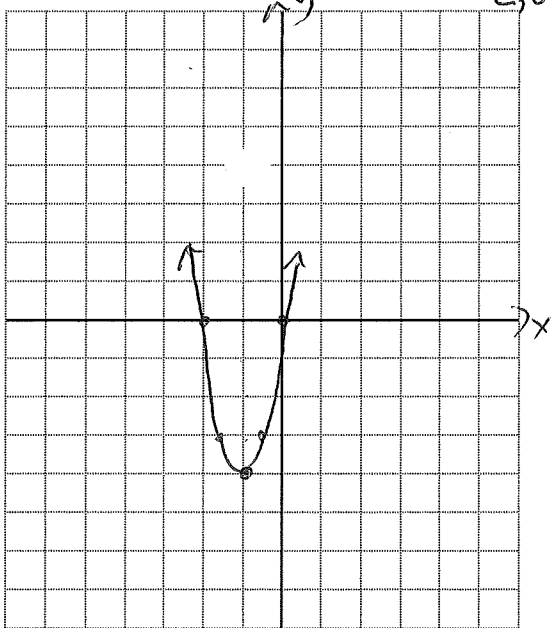
$(0,0) \rightarrow (-1,-4)$

$(1,1) \rightarrow (-0.5,-3)$

$(2,4) \rightarrow (0,0)$

$(-1,1) \rightarrow (-1.5,-3)$

$(-2,4) \rightarrow (-2,0)$



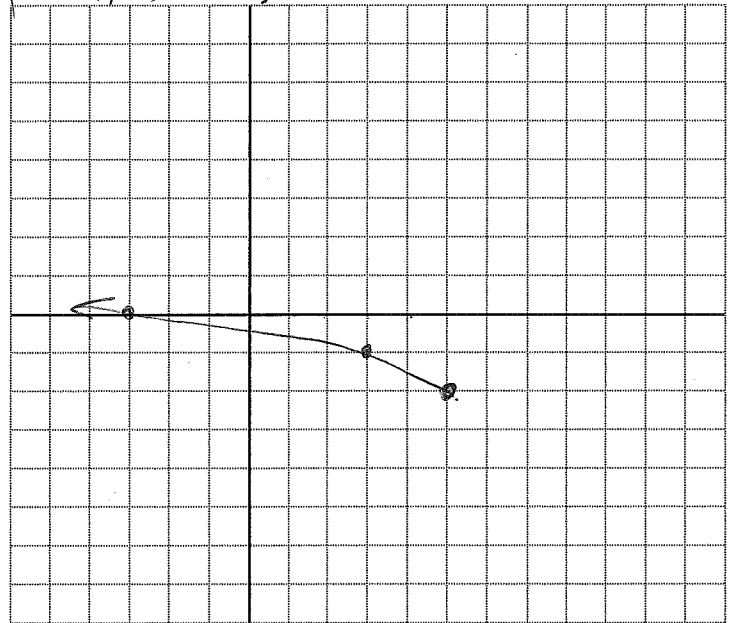
b) Graph $y = \sqrt{-\frac{1}{2}(x-5)} - 2$

$(4,2) \rightarrow (-3,0)$

$(x,y) \rightarrow (-2x+5, y-2)$

$(0,0) \rightarrow (5,-2)$

$(1,1) \rightarrow (3,-1)$



c) Graph $y = -3|x+2| + 5$

$(x,y) \rightarrow (x-2, -3y+5)$

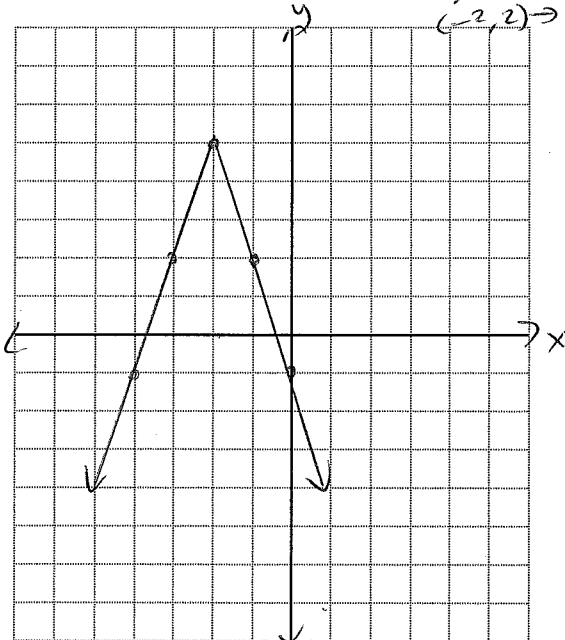
$(0,0) \rightarrow (-2,5)$

$(1,1) \rightarrow (-1,2)$

$(-1,1) \rightarrow (-3,2)$

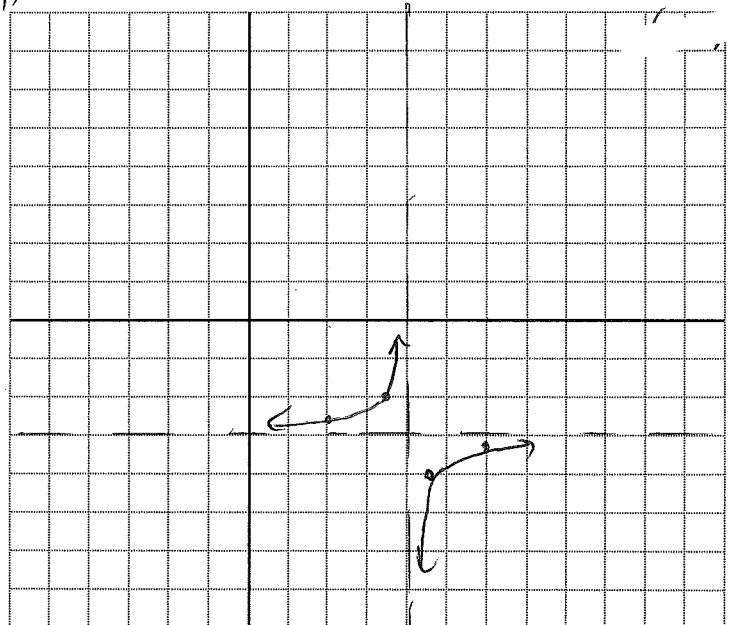
$(2,2) \rightarrow (0,-1)$

$(-2,2) \rightarrow (-4,-1)$



d) Graph $y = \frac{-1}{2(x-4)} - 3$

$(x,y) \rightarrow (\frac{x}{2}+4, -y-3)$



$(1,1) \rightarrow (4.5,-4)$

$(4,0.25) \rightarrow (6.375,-3.5)$

$(-1,-1) \rightarrow (3.5,-2)$

$(-4,-0.25) \rightarrow (2.375,-3.75)$

Summary: Transforming Relations

Transformation of $y = f(x)$		Description of Transformation	Mapping
$y = -f(x)$		Reflection about x-axis	$(x, -y)$
$y = f(-x)$		Reflection about y-axis	$(-x, y)$
$y = a f(x)$	$ a > 1$	vertically stretched by a factor of $ a $.	(x, ay)
	$0 < a < 1$	vertically compressed by a factor of $ a $.	(x, ay)
$y = f(kx)$	$ k > 1$	horizontally compressed by a factor of $\frac{1}{ k }$	$(\frac{x}{k}, y)$
	$0 < k < 1$	horizontally stretched by factor of $\frac{1}{ k }$	$(\frac{x}{k}, y)$
$y = f(x) + c$	$c > 0$	translation 'c' units up	$(x, y+c)$
	$c < 0$	translation c units down	$(x, y+c)$
$y = f(x - d)$	$d > 0$	translations right 'd' units.	$(x+d, y)$
	$d < 0$	translate left d units.	$(x+d, y)$
$y = a f[k(x - d)] + c$		Reflections Stretches Translations	$(\frac{x}{k} + d, ay + c)$