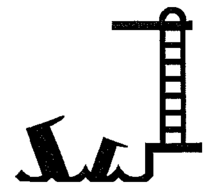
Day 7: Solve Problems Using Quadratic Equations

Warm-Up: The height, *h* metres, of a springboard diver above the surface of the water *t* seconds after he leaves the board is given by $h = -5t^2 + 10t + 3$.



a) How high was the diving board?

Set $t=0$

Solve for h (find h)

$h = -5(0)^2 + 10(0) + 3$

$= 3$ m

b) When does he hit the water? Set $h=0$

$-5t^2 + 10t + 3 = 0$

$$t = \frac{-10 \pm \sqrt{10^2 - 4(-5)(3)}}{2(-5)}$$

$$= \frac{-10 \pm \sqrt{160}}{-10}$$

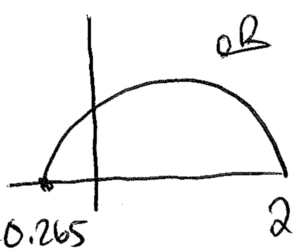
$$t_1 = \frac{-10 + 12.65}{-10} \qquad t_2 = \frac{-10 - 12.65}{-10}$$

$= -0.265$ (inadmissible) $\quad = 2.265$

c) Determine the diver's maximum height above the water.

Complete the square

OR



$$t = \frac{-0.265 + 2.265}{2}$$

$= 1$

sub $t=1$ $\boxed{h = \underline{8\text{ m}}}$

d) For how long is he above a height of 5 m?

$h = 5$

$5 = -5t^2 + 10t + 3$

$-5t^2 + 10t - 2 = 0 \Rightarrow 5t^2 - 10t + 2 = 0$

$$t = \frac{10 \pm \sqrt{(-10)^2 - 4(5)(2)}}{10}$$

$$= \frac{10 \pm \sqrt{60}}{10}$$

$t_1 = 1.77$

$t_2 = 0.23$

$\therefore 1.77 - 0.23 = 1.54$

1. An equation representing the height of a flare, h metres, above the release position, after t seconds, is $h = -5t^2 + 100t$.
   a. What is the height of the flare after 3 s? (255 m)
   b. What is the maximum height reached by the flare? (500 m)
   c. What is the height of the flare after 25 s? (-625 m)
   d. Does your answer in part c make sense? Explain. (No . . .)
   e. Determine the time for which the flare is higher than 80 m. (18.3 s)

a. $h = -5(3)^2 + 100(3)$
$= -5(9) + 300$
$= -45 + 300$
$= 255m$
∴ It's 255 m.

b. $h = -5(t^2 - 20t)$ $\frac{-20}{2} = -10$
$= -5(t^2 - 20t + 100 - 100)$ $(-10)^2 = 100$
$= -5(t^2 - 20t + 100) + 500$
$= -5(t-10)^2 + 500$
∴ the max height is 500m.

c) $h = -5(25)^2 + 100(25)$
$= -625m$

d) It reaches the ground before 25 sec.

e) $80 = -5t^2 + 100t$
$5t^2 - 100t + 80 = 0$ GCF = 5
$\frac{5(t^2 - 20t + 16)}{5} = \frac{0}{5}$ divide each side by 5
$t^2 - 20t + 16 = 0$

Use quadratic formula
$t^2 - 20t + 16 = 0$
$a = 1$ $b = -20$ $c = +16$
$x_{1,2} = \frac{-(-20) \mp \sqrt{(-20)^2 - 4(1)(16)}}{2(1)}$

$x_1 = \frac{20 + \sqrt{400-64}}{2} = \frac{20 + 18.3}{2} \doteq 19.2$

$x_2 = \frac{20 - \sqrt{400-64}}{2} = \frac{20 - 18.3}{2} = 0.85$

∴ $19.2 - 0.85 = 18.3$ sec.

2. When a flare is fired vertically upward, its height, h metres, after t seconds is modelled by the equation $h = -5t^2 + 153.2t$.
   a. Is the flare on the ground or on a stand? (ground)
   b. How long is the flare in the air? (30.64 sec)
   c. What is the maximum height of the flare? (1173.5 m)
   d. For how many seconds is the flare higher than 1 km. (11.78 s) → 1km = 1000m

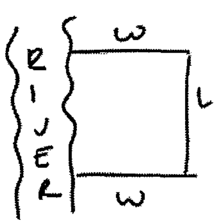
a. Std form gives the y-int.
$h = -5t^2 + 153.2t + 0$
y-int = 0, it's on the ground.

b. We need to find x-int.
$0 = -5t^2 + 153.2t$ GCF = $-5t$
$0 = -5t(t - 30.64)$
$-5t = 0$ $\boxed{t = 0}$
$t - 30.64 = 0$ $\boxed{t = 30.64}$
∴ It's in the air for 30.64 sec.

c. We can average the zeros
$x = \frac{0 + 30.64}{2} = 15.32$
$y = -5(15.32)^2 + 153.2(15.32)$
$\boxed{y = 1,173.5m.}$ max height
Vertex (15.32, 1173.5)

d.
$1000 = -5t^2 + 153.2t$
$5t^2 - 153.2t + 1000 = 0$
$a = 5$ $b = -153.2$ $c = 1000$
$x_{1,2} = \frac{-(-153.2) \mp \sqrt{(-153.2)^2 - 4(5)(1000)}}{2(5)}$
$x_{1,2} = \frac{153.2 \mp 58.9}{10}$
$x_1 = \frac{153.2 + 58.9}{10} = 21.21$
$x_2 = \frac{153.2 - 58.9}{10} = 9.43$
∴ $21.21 - 9.43 = 11.78$ s. it was above 1km.

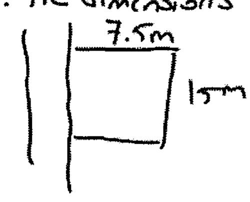
3. A rectangular lot is bounded on one side by a river and on the other three sides by a total of 30 m of fencing. A formula that represents the area of the lot, A square metres, in terms of its width, x metres, is $A = 30x - 2x^2$. Calculate the dimensions of the largest possible lot. (7.5 m by [redacted] m)



$30 = 2w + L$
$30 - 2w = L$

$A = w(30 - 2w)$
$= 30w - 2w^2$
$= -2w^2 + 30w$
$= -2(w^2 - 15w)$ $\frac{-15}{2}$ $(-\frac{15}{2})^2 = 56.25$
$= -2(w^2 - 15w + 56.25 - 56.25)$
$= -2(w^2 - 15w + 56.25) + 112.5$
$= -2(w - 7.5)^2 + 112.5$

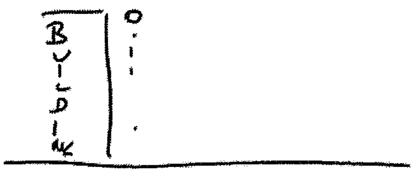
∴ the dimensions are



4. A ball is dropped over the roof of a building. The equation to model this scenario is: $h = -16t^2 + 75$, where h is the height of the building in feet after t seconds.

a. How high is the building? (75 ft)

b. How long does it take the ball to land? ([redacted] sec)



a). $h = -16t^2 + 75$ sub "0" for "t"

$= -16(0)^2 + 75$

$= 75$ ft.

b.) Solve the equation

$0 = -16t^2 + 75$ move $-16t^2$ to LS

$\frac{16t^2}{16} = \frac{75}{16}$

$\sqrt{t^2} = \sqrt{4.6875}$

$t \doteq \mp 2.17$

∴ It takes app. 2.17 sec.

5. The power, P watts, supplied to a circuit by a 9-V battery is given by the formula $P = 9I - 0.5I^2$, where I is the current in amperes. What is the maximum power? (40.5 W)

$P = -0.5I^2 + 9I$

$= -0.5(I^2 - 18I)$ $-18/2 = -9$ $(-9)^2 = 81$

$= -0.5(I^2 - 18I + 81 - 81)$

$= -0.5(I^2 - 18I + 81) + 40.5$

$= -0.5(I - 9)^2 + 40.5$

Vertex is $(9, 40.5)$; therefore the max power is 40.5 W

6. Computer software programs are sold to students for \$20 each. Three hundred students are willing to buy them at this price. For every \$5 increase in price, there are 30 fewer students willing to buy the software. A formula that represents the revenue, R dollars, for an x dollar increase in price is $R = -6x^2 + 180x + 6000$. Calculate the selling price that will produce the maximum revenue. What is the maximum revenue? (\$35, \$7350)

Revenue = Price × Amount

$= (20 + 5x)(300 - 30x)$

$= 6000 - 600x + 1500x - 150x^2$

$R = -150x^2 + 900x + 6000$

$= -150(x^2 - 6x) + 6000$ $\frac{-6}{2} = -3$ $(-3)^2 = 9$

$= -150(x^2 - 6x + 9 - 9) + 6000$

$= -150(x^2 - 6x + 9) + 1350 + 6000$

$= -150(x - 3)^2 + 7350$

∴ Vertex is $(3, 7350)$

price change

∴ When you set the price to $20 + 5(3) = \$35$, you will max the rev. to \$7350

7. When a baseball is hit at a certain velocity and angle the height of the ball is given by the equation $h = -0.0032x^2 + x + 3$, where h is the height of the ball in feet, and x is the horizontal distance from home plate in feet.
   a. How high was the ball when it was hit? (3 ft)
   b. How high is the ball when it is 2 ft away from home plate? (4.98 ft)
   c. How far away from home plate does the ball land? (315.47 ft)
   d. What is the maximum height reached by the baseball? (81.125 ft)

a) $h = -0.032x^2 + x + 3 \quad x=0$

$\boxed{h = 3\text{ft}}$

b) $h = -0.0032(2)^2 + 2 + 3$

$= 4.9872\text{ ft}$

c. $0 = -0.032x^2 + x + 3$

$a = -0.032 \quad b = 1 \quad c = 3$

$$x_{1,2} = \frac{-1 \mp \sqrt{1^2 - 4(-0.0032)(3)}}{2(-0.0032)} = \frac{-1 \mp \sqrt{1.0384}}{-0.0064}$$

$$x_1 = \frac{-1 + 1.0190}{-0.0064} \qquad x_2 = \frac{-1 - 1.0190}{-0.0064}$$

$x_1 \doteq -2.97 \qquad x_2 \doteq 315.47\text{ ft}$

∴ It lands 315.47ft away.

8. Forty metres of fencing are available to enclose a rectangular pen. The area, A square metres, enclosed is given by $A = \cancel{20x - x^2}$, where the length of the pen is x metres.
   e. What is the maximum area that can be enclosed? (100 m^2)
   f. What are the dimensions of the pen with the maximum area? (10 m by 10 m)
   g. What length produces a pen with an area greater than 90 m^2? (between 6.9 m and 13.1 m)

(a) L, w

$2(L + w) = 40$
$L + w = 20$
$L = 20 - w$

⇒ 20−w, w

Area $= w(20 - w)$
$= -w^2 + 20w$
$= -(w^2 - 20w)$
$= -(w^2 - 20w + 100 - 100)$
$= -(w - 10)^2 + 100$

Vertex $(10, 100)$
width area

a & b:
∴ The max area is 100m² when the dimensions are 10m by 10m

c. $90 = -w^2 + 20w$

$w^2 - 20w + 90 = 0$

$a = 1 \quad b = -20 \quad c = 90$

$$x_{1,2} = \frac{-(-20) \mp \sqrt{(-20)^2 - 4(1)(90)}}{2(1)} = \frac{20 \mp \sqrt{40}}{2}$$

$x_1 = \frac{20 + 6.3246}{2} \doteq 13.2 \qquad x_2 = \frac{20 - 6.3246}{2} \doteq 6.8$ ∴ 6.8m and 13.2m

9. A company manufactures and sells designer T-shirts. The profit, P dollars, for selling a certain style of T-shirt is projected to be $P = -20x^2 + 1000x - 6720$, where x dollars is the selling price of one T-shirt.
   a. What are the break even points? (\$8 and \$42) when $P = 0$
   b. What selling price gives the maximum profit? What is the maximum profit? (\$25, \$5780)

$0 = -20x^2 + 1000x - 6720 \quad$ GCF $= -20$

$$\frac{0}{-20} = \frac{-20(x^2 - 50x + 336)}{-20}$$ divide each side by −20 to rid of −20

$0 = x^2 - 50x + 336$

$a = 1 \quad b = -50 \quad c = 336$

$$x_{1,2} = \frac{-(-50) \mp \sqrt{(-50)^2 - 4(1)(336)}}{2(1)}$$

$$x_{1,2} = \frac{50 \mp 34}{2} \quad x_1 = 42, \quad x_2 = 8$$

∴ The break even points are \$8 and \$42.

b. $P = -20(x^2 - 50x) - 6720 \qquad -50 \div 2 = -25 \quad (-25)^2 = 625$

$= -20(x^2 - 50x + 625 - 625) - 6720$

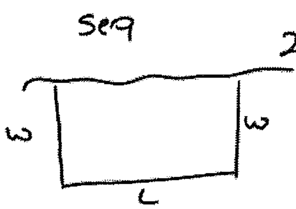
$= -20(x^2 - 50x + 625) + 12500 - 6720$

$= -20(x - 25)^2 + 5780$

Vertex is $(25, 5780)$

∴ The max profit is \$5780 when the price is \$25

10. A life guard marks a rectangular swimming area at a beach with a 200 m rope. The width of the swimming area is x metres. The area enclosed is A square metres, where $A = x(200 - 2x)$. What is the greatest area that can be enclosed? (5000 m^2)



$200 = 2w + L$

$200 - 2w = L$

$\text{Area} = w(200 - 2w)$

$= -2w^2 + 200w$

$= -2(w^2 - 100w)$ $\quad -100 \div 2 = -50$, $(-50)^2 = 2500$

$= -2(w^2 - 100w + 2500 - 2500)$

$= -2(w^2 - 100w + 2500) + 5000$

$= -2(w - 50)^2 + 5000$

Vertex is (50, 5000); therefore, the max area is 5000m^2 with 50m and 100m dimensions.

11. A company manufactures and sells novelty caps. The profit, P dollars, for selling a certain style of cap at t dollars each is projected to be $P = -15t^2 + 90t + 675$. What selling price is expected to give a maximum profit? What is the maximum profit? (\$3, \$810)

$P = -15(t^2 - 6t) + 675$ $\quad -6/2 = -3$, $(-3)^2 = 9$

$= -15(t^2 - 6t + 9 - 9) + 675$

$= -15(t^2 - 6t + 9) + 135 + 675$

$= -15(t - 3)^2 + 810$

V(3, 810) $\therefore$ When the price is \$3, the max profit is \$810

12. A stone is thrown upward with an initial speed of 25 m/s. Its height, h metres, after t seconds is given by the equation $h = -5t^2 + 25t$. For how long is the stone higher than 30 m? (1 sec)

$30 = -5t^2 + 25t$

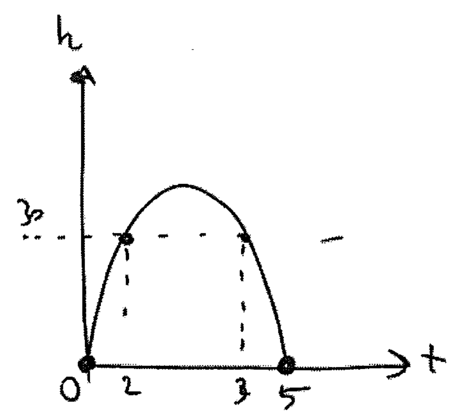
$0 = -5t^2 + 25t - 30$

$0 = -5(t^2 - 5t + 6)$

$0 = -5(t - 2)(t - 3)$

$t - 2 = 0$ $\quad$ $t - 3 = 0$

$\boxed{t = 2}$ $\quad$ $\boxed{t = 3}$



$\therefore 3 - 2 = 1$ sec.