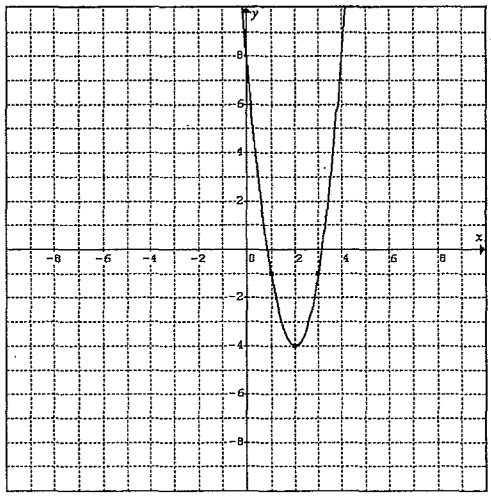
MPM2D1 — Date: ______

Day 7: Equations of Parabolas — Chapter 4: Quadratic Relations

WHAT IS THE EQUATION?

Determine the equation in vertex form: $y = a(x-h)^2 + k$



The vertex is $(2,-4)$

Another point is $(1,-1)$

$y = a(x-2)^2 - 4$

sub $x = 1$. $y = -1$

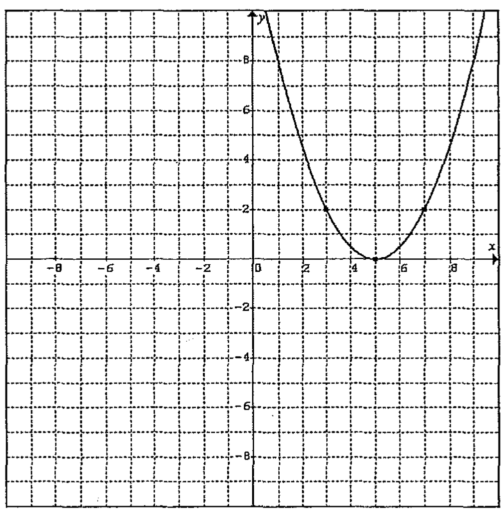
$-1 = a(1-2)^2 - 4 \Rightarrow -1 = a(-1)^2 - 4$

$-1 = a - 4$

$a = -1 + 4$

$= 3$

$\therefore y = 3(x-2)^2 - 4$



The vertex is $(5,0)$

Another point is $(7,2)$

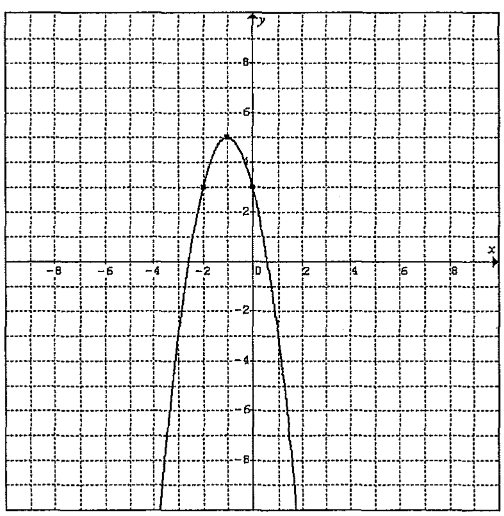
$y = a(x-5)^2 + 0$ sub $x = 7$, $y = 2$

$2 = a(7-5)^2 + 0$

$2 = 4a$

$a = \frac{2}{4} = \frac{1}{2}$

$\therefore y = \frac{1}{2}(x-5)^2$



The vertex is $(-1,5)$

Another point is $(0,3)$

$y = a(x+1)^2 + 5$

$3 = a(0+1)^2 + 5$

$3 = a(1)^2 + 5$

$3 = a + 5$

$a = 3 - 5$

$= -2$

$\therefore y = -2(x+1)^2 + 5$

The vertex is (4, -2). Another point is (7, 1). $y = a(x-4)^2 - 2$ $1 = a(7-4)^2 - 2$ $1 = 9a - 2$ $3 = 9a$ $a = \frac{3}{9} = \frac{1}{3}$ $\therefore y = \frac{1}{3}(x-4)^2 - 2$	The vertex is (2, 0). Another point is (0, -2). $y = a(x-2)^2 + 0$ $-2 = a(0-2)^2 + 0$ $-2 = a(-2)^2 + 0$ $-2 = 4a$ $a = \frac{-2}{4} = \frac{-1}{2}$ $\therefore y = \frac{-1}{2}(x-2)^2$
The vertex is (0, -3). Another point is (1, -4). $y = a(x-0)^2 - 3$ $-4 = a(1-0)^2 - 3$ $-4 = a(1)^2 - 3$ $-4 = a - 3$ $a = -4 + 3$ $= -1$ $\therefore y = -(x)^2 - 3$	The vertex is (-3, -4). Another point is (-2, 1). $y = a(x+3)^2 - 4$ $1 = a(-2+3)^2 - 4$ $1 = a(1)^2 - 4$ $1 = a - 4$ $a = 1 + 4$ $= 5$ $\therefore y = 5(x+3)^2 - 4$
The vertex is (-4, 8). Another point is (0, 0). $y = a(x+4)^2 + 8$ $0 = a(0+4)^2 + 8$ $0 = 16a + 8$ $a = \frac{-8}{16} = \frac{-1}{2}$ $\therefore y = \frac{-1}{2}(x+4)^2 + 8$	The vertex is (5, 1). Another point is (1, 5). $y = a(x-5)^2 + 1$ $5 = a(1-5)^2 + 1$ $5 = a(-4)^2 + 1$ $5 = 16a + 1$ $4 = 16a$ $a = \frac{4}{16} = \frac{1}{4}$ $\therefore y = \frac{1}{4}(x-5)^2 + 1$

1. Find the equation of the parabola with vertex (0,-6), opening down and a vertical compression factor of 1/3 .

   $y = -\frac{1}{3}(x)^2 - 6$

2. Find the equation of the parabola with vertex (0,4), opening down and vertical stretch by a factor of 2.

   $y = -2(x)^2 + 4$

3. Find the equation of the parabola compressed vertically by a factor of one-quarter, and then translated 4 units to the right and one unit up.

   $y = \frac{1}{4}(x-4)^2 + 1$

4. What happens to the point (3,9) on the graph of $y=x^2$ when the parabola is reflected about the x-axis then stretched vertically by a factor of two?

   (3, 9) will be (3, -9) after reflection.
   (3, -9) will be (3, -18) after stretch.
   $\therefore (3,9) \rightarrow (3,-18)$.

5. Find the value of k so that the parabola $y = -\frac{1}{3}x^2 + k$ passes through (6,8).

   $8 = -\frac{1}{3}(6)^2 + k$
   $8 = -\frac{1}{3}(36) + k$
   $8 = -12 + k \Rightarrow k = 20$

6. Find the value of a and k so that the parabola passes through the points (1,-1) and (2,5)
   The parabola is in the form $y = ax^2 + k$

   $5 = 4a + k \rightarrow$ sub $x = 2$ $y = 5$
   $-1 = a + k \rightarrow$ sub $x = 1, y = -1$
   $- \quad 6 = 3a$
   $a = 2 \Rightarrow$ sub $a = 2$ in $-1 = a + k$
   $k = -3$
   $\boxed{\therefore a = 2 \quad k = -3}$

7. Write an equation for the parabola with a vertex $(-5,-3)$ passing through $(-3,-11)$.

   $y = a(x+5)^2 - 3$
   sub $x = -3$, $y = -11$
   $-11 = a(2)^2 - 3$
   $-11 = 4a - 3$
   $-8 = 4a$
   $a = -2$
   $\therefore y = -2(x+5)^2 - 3$

Homework: p. 185 #1-5