MPM2D

Day 6: **Quadratic Formula**

A root is the value of the variable that makes an equation true. It is the same as the solution to an equation.

Recall that a quadratic equation is an equation of the form

$ax^2+bx+c=0$.

If $y = 3x^2 - 6x + 11$, find the x value that makes $y = 10$.

$10 = 3x^2-6x+11$

$3x^2-6x+11-10=0$

$3x^2-6x+1=0$ (we can not factor)

Use QF $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=3 \quad b=-6 \quad c=1$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2-4(3)(1)}}{2(3)}$

$= \dfrac{6 \pm \sqrt{36-12}}{6}$

$= \dfrac{6 \pm \sqrt{24}}{6}$

$x_1 = \dfrac{6+\sqrt{24}}{6}$ OR $\dfrac{6-\sqrt{24}}{6}$

$\doteq 1.82 \quad \doteq 0.18$

Exact answers:

$x = \dfrac{6 \pm \sqrt{4}\sqrt{6}}{6} = \dfrac{6 \pm 2\sqrt{6}}{6}$

$= \dfrac{3+\sqrt{6}}{3}$

What happens if a quadratic equation cannot be factored?

Use quadratic formula to solve for x.

To determine x for a quadratic equation of the form $ax^2+bx+c=0$, we can use the quadratic formula to find the roots:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Yes, it is ugly, but you **must** memorize this formula!

You may for once, use a calculator!

Example 1 Use the quadratic formula to solve each quadratic equation. Where necessary, round to four decimal places.

a) $0 = 2x^2 + 9x + 6$

$a=2 \quad b=9 \quad c=6$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(2)(6)}}{2(2)}$$

$$= \frac{-9 \pm \sqrt{81-48}}{4}$$

$$x = \frac{-9 \pm \sqrt{33}}{4}$$

$$x_1 = \frac{-9+\sqrt{33}}{4} \doteq -0.8139$$

$$x_2 = \frac{-9-\sqrt{33}}{4} \doteq -3.6861$$

b) $4x^2 - 12x = -9$

$4x^2 - 12x + 9 = 0$

$a=4 \quad b=12 \quad c=9$

$$x = \frac{+12 \pm \sqrt{144 - 4(4)(9)}}{2(4)}$$

$$= \frac{+12 \pm \sqrt{144-144}}{8}$$

$$= \frac{+12 \pm \sqrt{0}}{8} = \frac{+12}{8} = +\frac{3}{2} = 1.5$$

$\therefore x = +\frac{3}{2}$ (order 2)

↳ (which implies perfect square trinomial)

$4x^2 - 12x + 9 = (2x-3)^2$

Example 2 Find the x-intercepts, the vertex, and the equation of the axis of symmetry of the quadratic relation $y = -3 + 8x - 5x^2$. Sketch the parabola.

Sub $y=0$ for x-ints.

$-5x^2 + 8x - 3 = 0$

$a=-5 \quad b=8 \quad c=-3$

$$x = \frac{-8 \pm \sqrt{64 - 4(-5)(-3)}}{2(-5)}$$

$$= \frac{-8 \pm \sqrt{4}}{-10}$$

$$= \frac{-8 \pm 2}{-10}$$

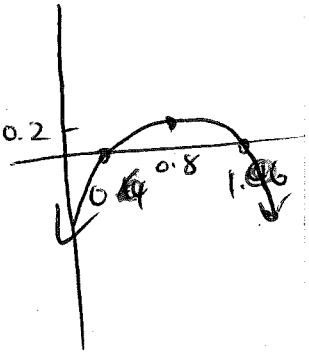
$$x_1 = \frac{-8+2}{-10} = \frac{-6}{-10} = 0.6$$

$x_2 = 1$

$$x_V = \frac{x_1 + x_2}{2} = \frac{0.6+1}{2} = 0.8$$

$y_V = -3 + 8(0.8) - 5(0.8)^2 = 0.2$

$\therefore$ vertex $(0.8, 0.2)$



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