Day6-MCR3U

Sine Law – The Ambiguous Case

Ambiguous is defined as "having more than one possible meaning". In trigonometry, ambiguity exists when we try to solve some SSA triangles (Sine Law).

When considering triangles where $\angle A$ is obtuse: there is one solution if $a > b$, and no solutions for $a \leq b$ (consider that $\angle A > 90°$, so it is the largest angle, which means that a must be the longest side).

An ambiguous case is only possible if $\angle A$ is acute (i.e.)

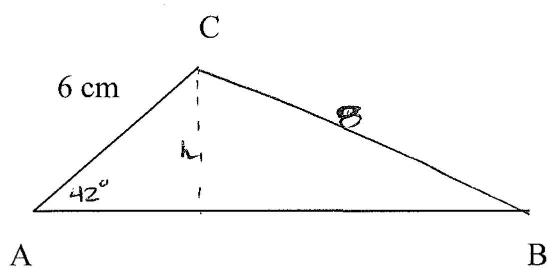
TASK: In each case below, determine the value of $\angle B$ (if possible).

Case 1

If $\angle A$ is acute and $a \geq b$, there is only 1 solution because only one triangle can be created. ***Therefore, no ambiguity exists.***

since $a > h$, 1 sol'n.
$a > b$

Ex. Given $\angle A = 42°$, $b = 6$ cm, $a = 8$ cm.



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{b \sin A}{a}$$

$$= \frac{6 \sin 42}{8} \doteq 0.502$$

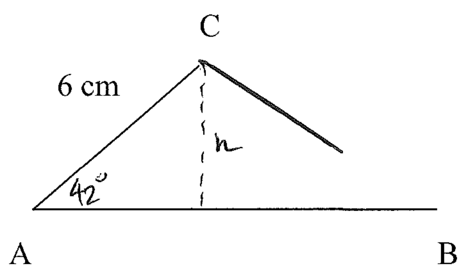
$$\boxed{\angle B \doteq 30°}$$

$h = b \sin A$
$= 6(\sin 42°)$
$= 4.014$

Case 2

If $\angle A$ is acute and $a < b \sin A$, there is 0 solution because a triangle cannot be drawn..

Ex. Given $\angle A = 42°$, $b = 6$ cm, $a = 3$ cm.



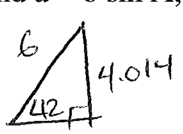
since $a < h$, there is no triangle that can be drawn.

h is the height (the shortest distance which means a has to be at least 4.014 (4.014 or more)

$h = b \sin A$
$= 6 \sin 42$
$= 4.014$

Case 3

If $\angle A$ is acute and $a = b \sin A$, there is 1 solution – a right angled triangle. (Example: $a = 4.015$ cm)

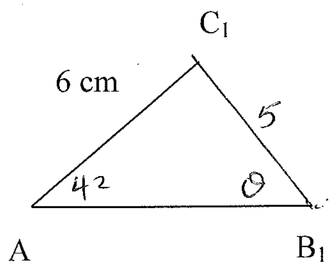


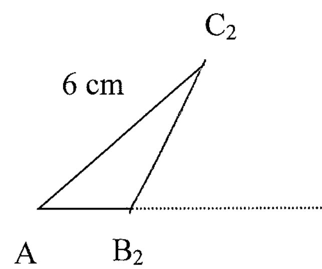
$\angle B = 90°$.

Case 4

If $\angle A$ is acute and $b \sin A < a < b$, there are _____ solutions since two possible triangles can be drawn.
Therefore, an ambiguous case exists.

Ex. Given $\angle A = 42°$, $b = 6$ cm, $a = 5$ cm.





$\beta_2 = 180 - 53 = 127°$

$\frac{\sin\theta}{6} = \frac{\sin 42}{5}$

$\sin\theta = \frac{6\sin 42}{5}$

$\theta = 53°$ $\beta_1 = 53°$

Summary

$\angle A > 90°$: $a > b$: 1 solution(s) $a \le b$: 0 solution(s)

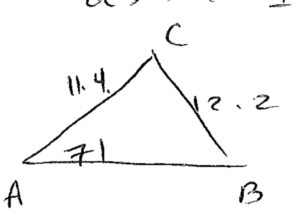
$\angle A < 90°$: $a \ge b$: 1 solution(s) $a < b \sin A$: 0 solution(s)

$a = b \sin A$: 1 solution(s) $b \sin A < a < b$: 2 solution(s)

Practice: For each triangle, determine the number of solutions. Solve the triangle.

a) ΔABC: $\angle A = 71°$, $a = 12.2$, $b = 11.4$ (1 soln, $\angle B = 62°$, $\angle C = 47°$, $c = 9.4$)

$a > b \therefore$ 1 triangle.



$\frac{\sin B}{11.4} = \frac{\sin 71}{12.2}$

$\sin B = \frac{(11.4)(\sin 71)}{12.2}$

$\angle B = 62$

$\angle C = 180 - 71 - 62 = 47°$

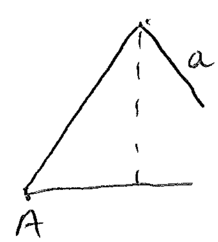
$\frac{c}{\sin C} = \frac{a}{\sin A}$

$c = \frac{12.2 \sin 47}{\sin 71} \doteq 9.4$

b) ΔABC: $\angle A = 55°$, $a = 7.1$, $b = 9.6$ (0 solutions)

$h = b \sin A$
$= 9.6 \sin 55$
$= 7.86$

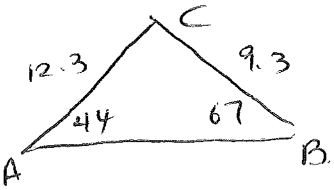
$a < b \sin A \Rightarrow$ 0 solutions



c) $\triangle ABC$: $\angle \boldsymbol{A} = 44°$, a = 9.3, b = 12.3 (2 solns, $\angle \boldsymbol{B} = 67°$, $\angle \boldsymbol{C} = 69°$, c = 12.5 OR $\angle \boldsymbol{B} = 113°$, $\angle \boldsymbol{C} = 23°$, c = 5.2)

$b\sin A = 12.3\sin 44 = 8.54$

$h < a < b$ implies ambiguous case.



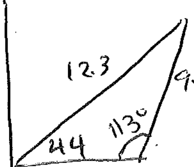
$\dfrac{\sin B}{12.3} = \dfrac{\sin 44}{9.3}$

$\angle B = 67°$.

$\angle C = 69°$

$\dfrac{c}{\sin 69} = \dfrac{9.3}{\sin 44}$

$c \doteq 12.5$



$180 - 67 = 113$

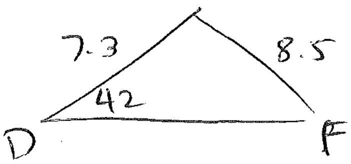
$\angle C = 180 - 44 - 113 \doteq 23°$

$\dfrac{c}{\sin 23} = \dfrac{12.3}{\sin 113}$

$c \doteq 5.2$

d) $\triangle DEF$: $\angle \boldsymbol{D} = 42°$, d = 8.5, f = 7.3 (1 soln, $\angle \boldsymbol{E} = 103°$, $\angle \boldsymbol{F} = 35°$, e = 12.4)

$d > f$ (and $d > h$)



$\dfrac{\sin F}{7.3} = \dfrac{\sin 42}{8.5}$

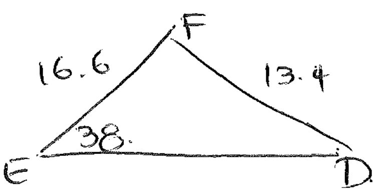
$\angle F = 35°$.

$\angle E = 180 - 42 - 35 = 103°$

$\dfrac{e}{\sin 103} = \dfrac{8.5}{\sin 42} \Rightarrow e \doteq 12.4$

e) $\triangle DEF$: $\angle \boldsymbol{E} = 38°$, d = 16.6, e = 13.4 (2 solns, $\angle \boldsymbol{D} = 50°$, $\angle \boldsymbol{F} = 92°$, f = 21.8 ***OR*** $\angle \boldsymbol{D} = 130°$, $\angle \boldsymbol{F} = 12°$, f = 4.5)

$h < e < d \Rightarrow$ 2 triangles.



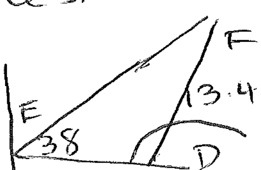
$\dfrac{\sin D}{16.6} = \dfrac{\sin 38}{13.4}$

$\angle D = 50°$

$\angle F = 180 - 50 - 38 = 92°$

$\dfrac{f}{\sin 92} = \dfrac{13.4}{\sin 38}$

$f \doteq 21.8$.



$180 - 50 = 130$

$\angle F = 180 - 130 - 38 = 12°$

$\dfrac{f}{\sin 12} = \dfrac{13.4}{\sin 38}$.

$f \doteq 4.5$.

f) $\triangle DEF$: $\angle \boldsymbol{D} = 71°$, d = 8.1, e = 12.2 (0 solutions)

$h = e\sin D$
$= 12.2(\sin 71)$
$= 11.5$

$d < h$ $\therefore$ No triangle can be drawn.

Homework: p. 318 #2-5, 11, 13, 14