

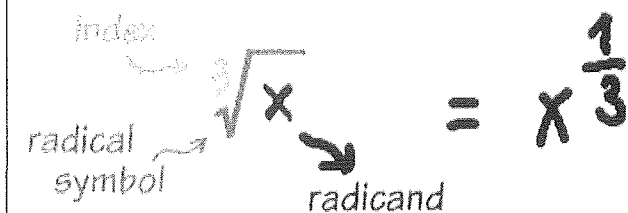
**What is a radical?**

An expression that has a square root, cube root, etc.  
The symbol is  $\sqrt{\quad}$

**Types of Radicals**

An entire radical is a radical with a coefficient of 1  
(e.g.  $\sqrt{40}$ )

A mixed radical has a coefficient other than 1  
(e.g.  $2\sqrt{5}$ ). It is 2 times  $\sqrt{5}$ .



Cube root of "x"

**A. SIMPLIFYING (Reducing) RADICALS**

To simplify means to find another expression with the same value.

It does not mean to find a decimal approximation.

i) Simplify the following "entire" radicals.

a)  $\sqrt{40} = \sqrt{4} \sqrt{10}$   
 $= 2\sqrt{10}$

b)  $\sqrt{72} = \sqrt{36} \sqrt{2}$   
 $= 6\sqrt{2}$

c)  $\sqrt{180}$   
 $= \sqrt{36} \sqrt{5}$   
 $= 6\sqrt{5}$

d)  $\sqrt{288} = \sqrt{144} \sqrt{2}$   
 $= 12\sqrt{2}$

ii) Express each of the following as "entire" radicals.

a)  $7\sqrt{5}$   
 $= \sqrt{49} \sqrt{5}$   
 $= \sqrt{195}$

b)  $-3\sqrt{3}$   
 $= -\sqrt{9} \sqrt{3}$   
 $= -\sqrt{27}$

Multiply or divide, then simplify the following radicals

a)  $2\sqrt{18} \times 3\sqrt{8}$   
 $= 6\sqrt{18} \cdot \sqrt{8}$   
 $= 6 \sqrt{9} \sqrt{2} \sqrt{4} \sqrt{2}$   
 $= (6)(3)(2)(\sqrt{2}\sqrt{2}) \cdot (2)$   
 $= 72$

b)  $5\sqrt{3} \times 7\sqrt{2}$   
 $= 35\sqrt{6}$

c)  $(\sqrt{3} + 5)^2$   
 $= (\sqrt{3} + 5)(\sqrt{3} + 5)$   
 $= 3 + 5\sqrt{3} + 5\sqrt{3} + 25$   
 $= 10\sqrt{3} + 28.$

d)  $\frac{-12\sqrt{24}}{3\sqrt{2}}$

$$= -4 \frac{\sqrt{12}\sqrt{2}}{\sqrt{2}}$$

$$= -4\sqrt{12} = -4\sqrt{4}\sqrt{3} \\ = -8\sqrt{3}$$

e)  $\frac{15}{\sqrt{5}}$  → no radical in the denominator  
multiply by  $\sqrt{5}$

$$= \frac{15}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{5} = 3\sqrt{5}$$

### D. ADDING RADICALS

When adding or subtracting radicals, you must use the same concept as that of adding or subtracting "like" variables. In other words, the radicals must be the same before you add (or subtract) them.

Like Radicals

example  $3\sqrt{6}$  and  $2\sqrt{6}$

non-example  $3\sqrt{6}$  and  $2\sqrt{5}$

Ex1: Add $3\sqrt{6} + 2\sqrt{6}$	Since the radicals are the same, simply add the numbers in front of the radicals (do NOT add the numbers under the radicals). Answer: $5\sqrt{6}$
Ex2: Add $3\sqrt{6} + 2\sqrt{5}$	Since the radicals are not the same, and both are in their simplest form, there is no way to combine these values. The answer is the same as the problem. Answer: $3\sqrt{6} + 2\sqrt{5}$

Add the following radicals

a)  $5\sqrt{3} + 2\sqrt{75}$

$$= 5\sqrt{3} + 2\sqrt{25}\sqrt{3} \\ = 5\sqrt{3} + 10\sqrt{3} \\ = 15\sqrt{3}$$

b)  $5\sqrt{8} - 3\sqrt{18} + \sqrt{3}$

$$= 5\sqrt{4}\sqrt{2} - 3\sqrt{9}\sqrt{2} + \sqrt{3} \\ = 10\sqrt{2} - 9\sqrt{2} + \sqrt{3} \\ = \sqrt{2} + \sqrt{3}$$

Practice: Multiplying Radicals

1. Simplify. Express your answer as a radical in simplest form.

a) $\sqrt{24} \times \sqrt{18}$ $= \sqrt{4} \sqrt{6} \sqrt{9} \sqrt{2}$ $= (2)(3) \sqrt{12}$ $= 6(\sqrt{4} \sqrt{3})$ $= 12\sqrt{3}$	b) $\sqrt{7} \times \sqrt{8}$ $= \sqrt{4} \sqrt{2} \sqrt{7}$ $= 2\sqrt{14}$	c) $\sqrt{11} \times \sqrt{14}$ $= \sqrt{154}$	d) $\sqrt{8} \times (-\sqrt{18})$ $= \sqrt{4} \sqrt{2} (-\sqrt{9} \sqrt{2})$ $= (2)(-3)(\cancel{2} \sqrt{2})$ $= -12$
e) $\sqrt{20} \times \sqrt{18}$ $= \sqrt{4} \sqrt{5} \sqrt{9} \sqrt{2}$ $= (2)(3)(\sqrt{10})$ $= 6\sqrt{10}$	f) $-\sqrt{32} \times \sqrt{72}$ $= -\sqrt{16} \sqrt{2} \sqrt{36} \sqrt{2}$ $= (-4)(6)(2)$ $= -48$	g) $\sqrt{20} \times \sqrt{32} \times \sqrt{18}$ $= \sqrt{4} \sqrt{5} \sqrt{16} \sqrt{2} \sqrt{9} \sqrt{2}$ $= (2)(4)(3)(2) \sqrt{5}$ $= 48\sqrt{5}$	h) $\sqrt{24} \times \sqrt{54} \times \sqrt{18}$ $= \sqrt{4} \sqrt{6} \sqrt{9} \sqrt{6} \sqrt{9} \sqrt{2}$ $= (2)(3)(6)(3) \sqrt{2}$ $= 108\sqrt{2}$

2. Simplify. Express your answer as a radical in simplest form.

a) $(3-\sqrt{2})(3+\sqrt{2})$ $= 9 + 3\sqrt{2} - 3\sqrt{2} - 2$ $= 9 - 2$ $= 7$	b) $(-7+\sqrt{7})(-7-\sqrt{7})$ $= 49 - 7$ $= 42$	c) $(\sqrt{11}-x)(\sqrt{11}+x)$ $= 11 - x^2$	d) $(\sqrt{5}-3)(3+\sqrt{5})$ $= (\sqrt{5}-3)(\sqrt{5}+3)$ $= 5 - 9$ $= -4$
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e) $(2+\sqrt{6})(3-\sqrt{10})$ $= 6 + 2\sqrt{10} + 3\sqrt{6} - \sqrt{60}$ $= 6 + 2\sqrt{10} + 3\sqrt{6} - 2\sqrt{15}$	f) $(-1-\sqrt{15})(8+\sqrt{15})$ $= -8 - \sqrt{15} - 8\sqrt{15} - 15$ $= -23 - 9\sqrt{15}$	g) $(x+1+\sqrt{2})(x+1-\sqrt{2})$ $= (x+1)^2 - (\sqrt{2})^2$ $= x^2 + 2x + 1 - 2$ $= x^2 + 2x - 1$	h) $(y-\sqrt{55})(y+\sqrt{22})$ $= y^2 - y\sqrt{55} - y\sqrt{55} - \sqrt{55}\sqrt{22}$ $= y^2 - 2y\sqrt{55} - 11\sqrt{10}$ $\sqrt{55}\sqrt{22} = \sqrt{5(\cancel{11})\sqrt{11}\sqrt{2}}$ $= 11\sqrt{10}$
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Homework:

1a) $12\sqrt{3}$	b) $2\sqrt{14}$	c) $\sqrt{154}$	d) $-12$
e) $6\sqrt{10}$	f) $-48$	g) $48\sqrt{5}$	h) $108\sqrt{2}$
2a) $7$	b) $42$	c) $11-x^2$	d) $-4$
e) $6+3\sqrt{6}-2\sqrt{10}-2\sqrt{15}$ f) $-23-9\sqrt{15}$ g) $x^2+2x-1$ h) $y^2+(\sqrt{22}-\sqrt{55})y-11\sqrt{10}$			

Puzzle Page

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