Day 4-MCR3U

Inverse of a Quadratic Function

Date: _____

Example 1:

a) Find the inverse of a $f(x) = x^2 - 1$.
b) Graph $f(x)$ and its inverse.
c) Is the inverse of $f(x)$ a function?
d) Determine the domain and range of $f(x)$ and its inverse.

a) $f(x) = x^2 - 1$

$y = x^2 - 1$

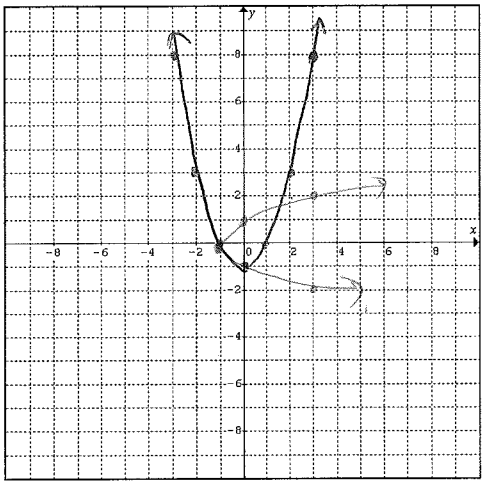
$x = y^2 - 1$

$x + 1 = y^2$

$y = \pm\sqrt{x+1}$

$f^{-1}(x) = \pm\sqrt{x+1}$

c) Inverse is not a function.
FAILS VLT.



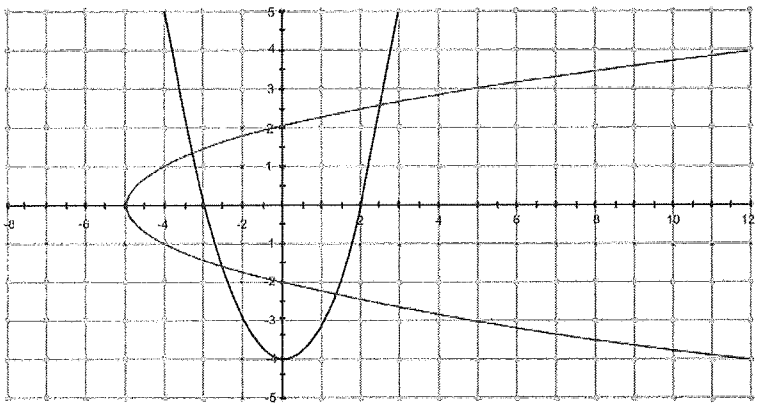
d) $D_f = \{x \in \mathbb{R}\}$

$R_f = \{y \in \mathbb{R} \mid y \geq -1\}$

$D_{f^{-1}} = \{x \in \mathbb{R} \mid x \geq -1\}$

$R_{f^{-1}} = \{y \in \mathbb{R}\}$

Example 2: The graph of f(x) and its inverse are shown below. Find an equation for f(x) and its inverse.



a) $f(x) = x^2 - 4$

$y = x^2 - 4$

$x = y^2 - 4$

$x + 4 = y^2$

$y = \pm\sqrt{x+4}$

$f^{-1}(x) = \pm\sqrt{x+4}$.

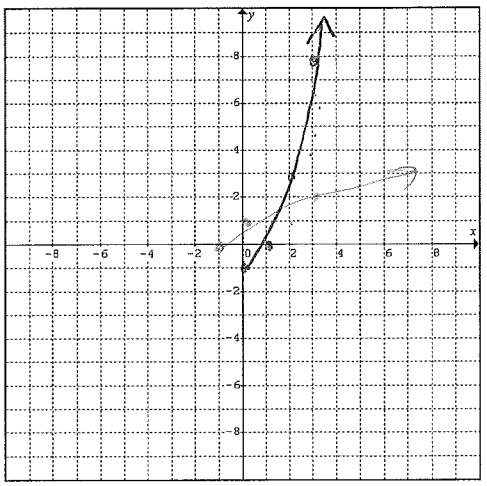
Example 3: Restrict the domain of each function so its inverse is also a function. Graph the function and its inverse.

a) $f(x) = x^2 - 1$

↳ restrict it to the right half or left half of parabola.

$y = x^2 - 1, \; x \geq 0$ | or $y = x^2 - 1, \; x \leq 0$

$f^{-1}(x) = \sqrt{x+1}$ | $f^{-1}(x) = -\sqrt{x+1}$



b) $f(x) = 2x^2 + 3$

$y = 2x^2 + 3, \quad x \geq 0$

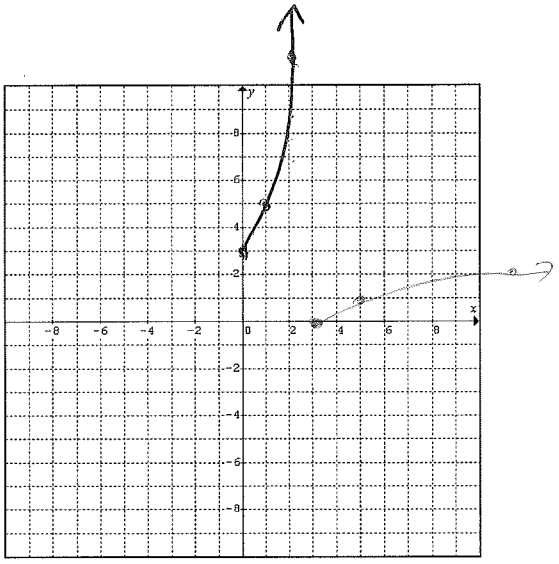
$x = 2y^2 + 3$

$x - 3 = 2y^2$

$\frac{x-3}{2} = y^2$

$\pm\sqrt{\frac{x-3}{2}} = y$ ↩ +ve since $x \geq 0$

$f^{-1}(x) = \sqrt{\frac{x-3}{2}}$



c) $f(x) = (x-1)^2$

(will restrict the domain to left half of the parabola)

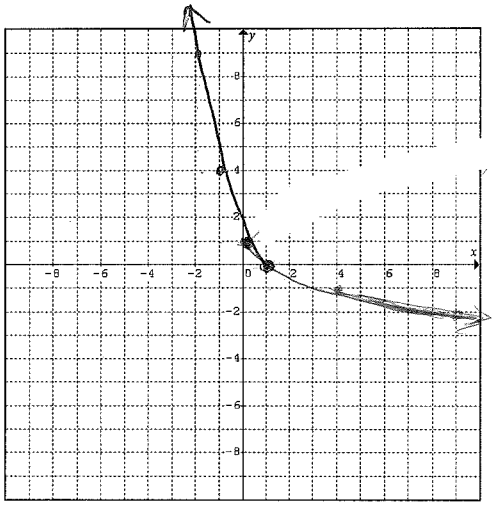
$y = (x-1)^2, \quad x \leq 1$

$x = (y-1)^2$

$\pm\sqrt{x} = y - 1$

$\pm\sqrt{x} + 1 = y \Rightarrow y = -\sqrt{x} + 1$

(since y has to be ≤ 1)



d) $f(x) = (x+2)^2 + 1$, $x \geq -2$ (Right half)

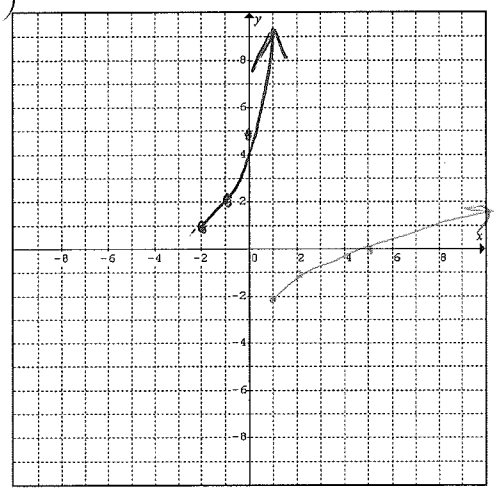
$y = (x+2)^2 + 1$

$x = (y+2)^2 - 1$

$x - 1 = (y+2)^2$

$\pm\sqrt{x-1} = y+2$ ↩ positive only

$+\sqrt{x-1} - 2 = y$



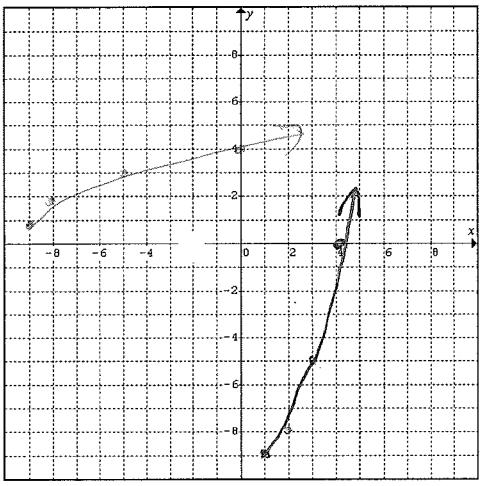
e) $f(x) = (x-4)(x+2)$

zeros: $4, -2$

$x_v = \frac{4+(-2)}{2} = 1$ sub in

$y_v = (-3)(3) = -9$ $y = (x-1)^2 - 9$

↓ X7/1



inverse: $x = (y-1)^2 - 9$

$x + 9 = (y-1)^2$

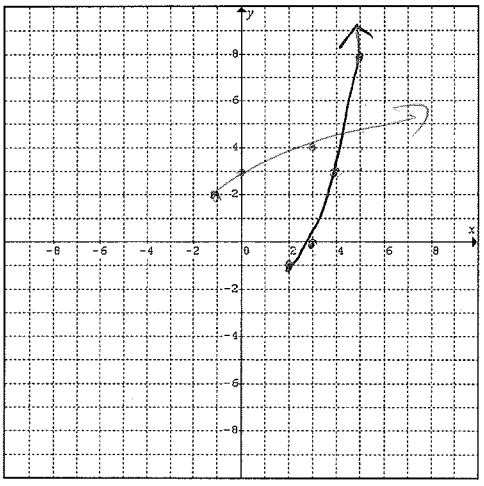
$\pm\sqrt{x+9} = y-1$

$\boxed{+\sqrt{x+9} + 1 = y}$

f) $f(x) = x^2 - 4x + 3$ → Complete the square first

$y = (x^2 - 4x + 4 - 4) + 3$

$= (x^2 - 4x + 4) - 4 + 3 = (x-2)^2 - 1$



INVERSE: $x = (y-2)^2 - 1$

$x + 1 = (y-2)^2$

$\pm\sqrt{x+1} = y-2$

$\pm\sqrt{x+1} + 2 = y$

$f(x) = (x-2)^2 - 1$

$x \geq 2$

$f^{-1}(x) = \sqrt{x+1} + 2$

Homework: p. 160 #2,3,4,7,13