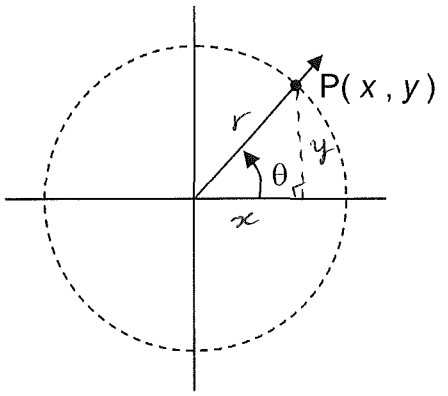
Day4-MCR3U

CAST Rule: Evaluating Trig Ratios for Any Angle Between 0° and 360°

Recall: the Standard Position of angles allows us to define the trigonometric ratios for ANY angle.



To determine the trig ratios for any angle in given a point P(x , y) on the terminal arm, we construct a right triangle by drawing a vertical line from the point P(x , y) to the x-axis.

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

Q1. For each diagram below:

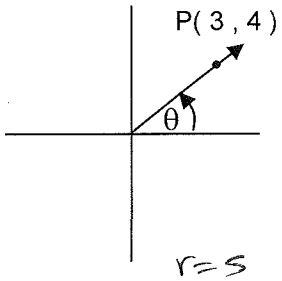
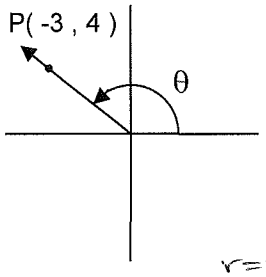
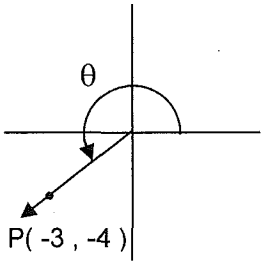
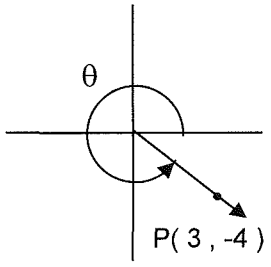
i. construct a right triangle by drawing a perpendicular line from P to the x-axis.

ii. determine the 'lengths' of all three sides of the right triangle constructed [include whether the length is positive or negative.]

iii. determine the three primary trig ratios: sine, cosine and tangent. [NOTE: why will some of these be negative?]

iv. Determine θ in degrees.

$3^2+(4)^2 = r^2 \Rightarrow r=5$

QUADRANT 1	QUADRANT 2	QUADRANT 3	QUADRANT 4
 r=5	 r=5		
$\sin\theta = \frac{y}{r} = \frac{4}{5}$	$\sin\theta = \frac{y}{r} = \frac{4}{5}$	$\sin\theta = \frac{y}{r} = \frac{-4}{5}$	$\sin\theta = \frac{y}{r} = \frac{-4}{5}$
$\cos\theta = \frac{x}{r} \quad \frac{3}{5}$	$\cos\theta = \frac{x}{r} = \frac{-3}{5}$	$\cos\theta = \frac{x}{r} = \frac{-3}{5}$	$\cos\theta = \frac{x}{r} = \frac{3}{5}$
$\tan\theta = \frac{y}{x} = \frac{4}{3}$	$\tan\theta = \frac{y}{x} = \frac{-4}{3}$	$\tan\theta = \frac{y}{x} = \frac{4}{3}$	$\tan\theta = \frac{y}{x} = \frac{-4}{3}$

Q2. In which quadrant is each primary trig ratio positive?

Q1: ALL

Q2: Sine

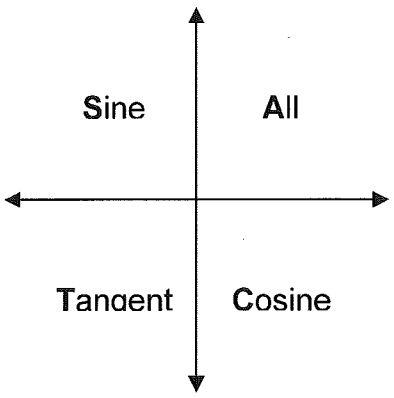
Q3: Tangent

Q4: Cosine

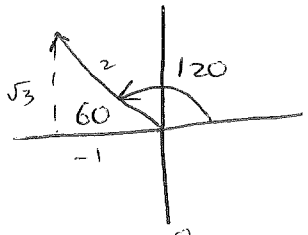
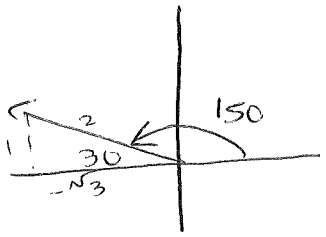
CAST Rule: Cosine – All – Sine – Tangent

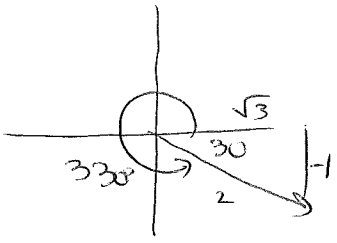
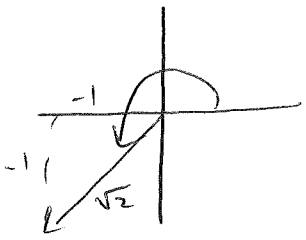
- indicates the quadrants where each trig ratio is POSITIVE.

tan − sin + cos −	tan + sin + cos +
tan + sin − cos −	tan − sin − cos +



Q3. Complete the following table.

	cos120°	**sin150°**
Is the trig ratio positive or negative?	Negative	Positive
Draw the terminal arm of the angle and determine its related angle.	 RAA = 60°	 RAA = 30°
What is the EXACT value of the trig ratio?	$\cos 120^\circ = -\frac{1}{2}$	$\sin 150 = \frac{1}{2}$

	tan330°	**Sin225°**
Is the trig ratio positive or negative?	Negative	Negative
Draw the terminal arm of the angle and determine its related angle.		
What is the EXACT value of the trig ratio?	$\tan 330^\circ = -\frac{1}{2}$	$\sin 225 = \frac{-1}{\sqrt{2}}$

11U

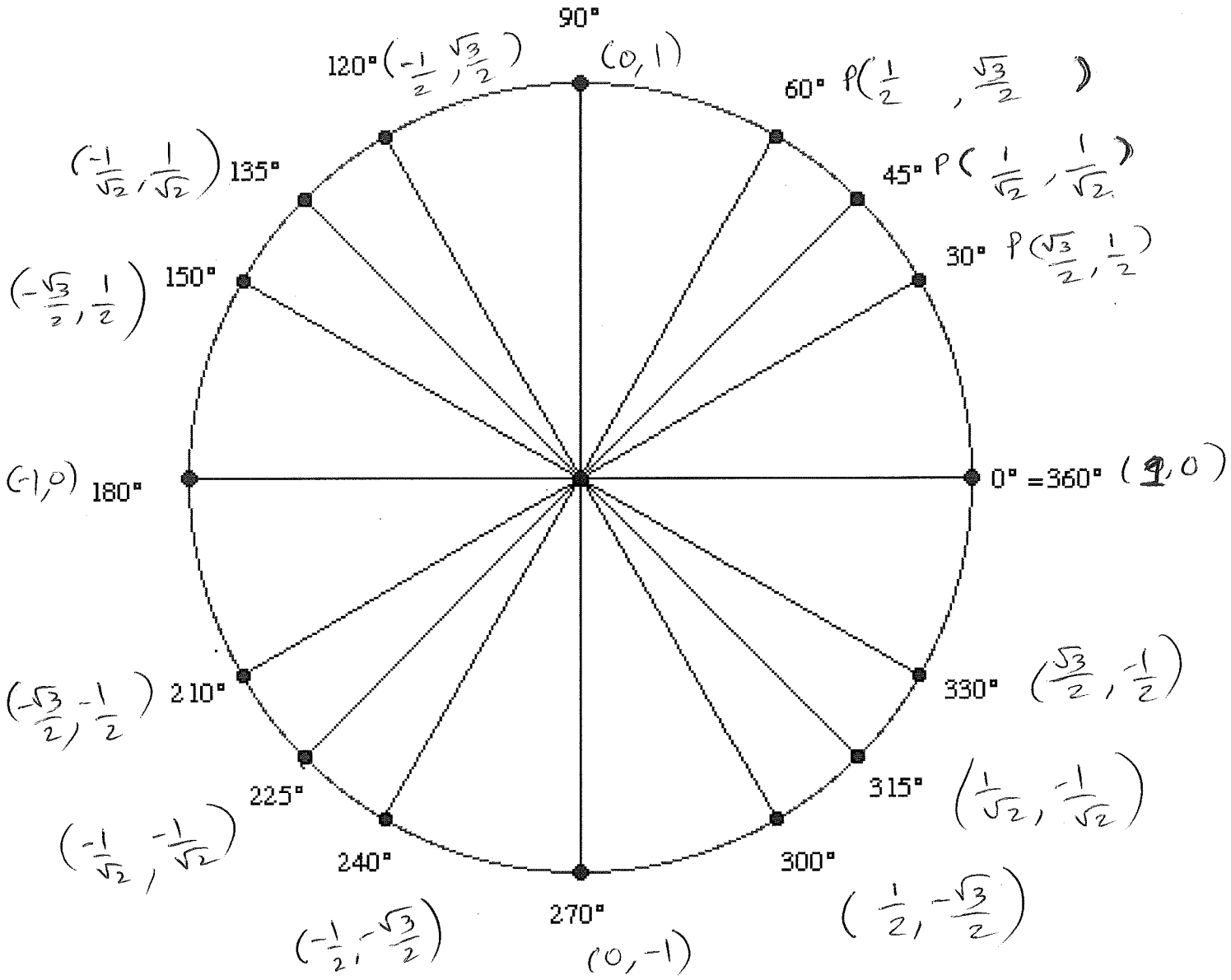
Unit Circle

$\sin\theta = \frac{y}{r}$

$y = r\sin\theta = \sin\theta \quad (r=1)$

$x = \cos\theta$

Date: _____



https://www.thinglink.com/scene/432267826410029057

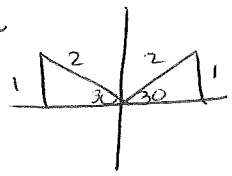
Q4. Determine the angle that represents the following:

a) $\sin\theta = \frac{1}{2}$ from the unit circle $\theta = 30, 150$

b) $\cos\theta = -\frac{\sqrt{3}}{2}$ from the unit circle $\theta = 150°, 210°$

c) $\tan\theta = -1$ $\theta = 135°$ or $315°$

OR

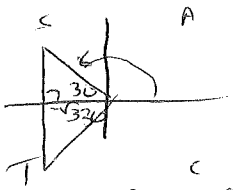


$\theta_1 = 30$

$\theta_2 = 180 - 30$

$= 50°$

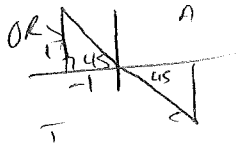
OR



$\therefore \theta_1 = 180 - 30 = 150°$

$\theta_2 = 180 + 30 = 210°$

OR



$\theta_1 = 180 - 45 = 135°$

$\theta_2 = 360 - 45 = 315°$

Q5. Determine the EXACT primary trig ratios for the following angles:

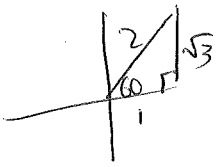
a) 60°

$\sin 60 = \frac{\sqrt{3}}{2}$

$\cos 60 = \frac{1}{2}$

$\tan 60 = \frac{\sqrt{3}}{2} \div \frac{1}{2}$

$= \sqrt{3}$

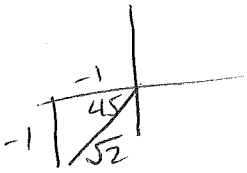


b) 225°

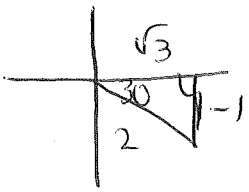
$\sin 225 = \frac{-1}{\sqrt{2}}$

$\cos 225 = -\frac{1}{2}$

$\tan 225 = \frac{-1}{\sqrt{2}} \div \frac{-1}{\sqrt{2}} = 1$



c) 330°

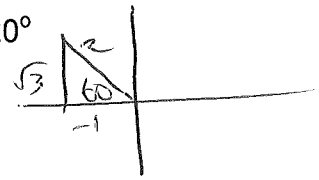


$\sin 330 = -\frac{1}{2}$

$\cos 330 = \frac{\sqrt{3}}{2}$

$\tan 330° = \frac{-1}{\sqrt{3}}$

d) 120°



$\sin 120 = \frac{\sqrt{3}}{2}$

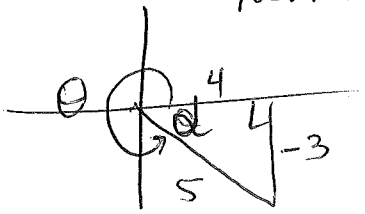
$\cos 120 = \frac{-1}{2}$

$\tan 120 = -\sqrt{3}$

Q6. For $\angle\theta$, $\sin\theta = \frac{-3}{5}$ and $\cos\theta = \frac{4}{5}$. Find $\angle\theta$.

↙ quadrant 3 or 4 → quadrant 1, 4

$\therefore \theta$ is in Q4



$\theta = 360 - \alpha$

$= 360 - 37 = 323°$

$\alpha = \sin^{-1}(3/5)$

or $\cos^{-1}(4/5)$

$= 37°$

COMPLETE: p. 299 #1, 2c,4c, 6bc, 8ace, 9f,14