

Recall: Evaluate

1)  $\frac{2}{3} \times \frac{15}{8}$

$$= \frac{(\cancel{2})(5)(\cancel{3})}{(\cancel{3})(\cancel{2})(4)} = \frac{5}{4}$$

2)  $\frac{2}{3} \div \frac{6}{7}$

$$= \frac{2}{3} \times \frac{7}{6}$$

$$= \frac{\cancel{2} \times 7}{3 \times 3 \times \cancel{2}} = \frac{7}{9}$$

Example 1 Simple Case

Simplify and state restrictions

$$\frac{2a^2}{5b^2c} \times \frac{3bc^2}{8a^2}$$

$$= \frac{(\cancel{2})(3) \cancel{a} \cdot \cancel{b} / \cancel{c}}{(5)(\cancel{2})(\cancel{4}) \cancel{a} \cdot \cancel{b} \cdot \cancel{b}}$$

$$= \frac{3c}{20b}, \quad a, b, c \neq 0$$

Example 2 Multiplying Rational Expressions

Simplify and state restrictions

$$\frac{x^2 - 9}{2x + 4} \times \frac{x^2 - 4}{x + 4x + 3}$$

$$= \frac{(x-3)(x+3)(x-2)(\cancel{x+2})}{2(x+\cancel{2})(x+3)(x+1)}$$

$$= \frac{(x-3)(x-2)}{2(x+1)}, \quad x \neq -3, -2, -1$$

1) Put brackets around numerator and denominator

2) Factor

3) State restrictions

4) A factor can be cancelled if it appears in the numerator and denominator

5) State final answer and restrictions again!

**\*Note: Always state restrictions before cancelling!!!**

**Example 3** Dividing Rational Expressions: Simplify and state restrictions

$$\frac{x^2 - x - 20}{x^2 - 5x} \div \frac{x^2 + 9x + 20}{x^2 - 12x + 36}$$

$$= \frac{(x-5)(x+4)}{(x)(x-5)} \div \frac{(x+4)(x+5)}{(x-6)(x-6)} \quad \leftarrow x \neq 0, 5, 6$$

$$= \frac{(x-5)(x+4)(x-6)(x-6)}{(x)(x-5)(x+4)(x+5)} \quad \leftarrow 2 \text{ more restrictions } x \neq -4, -5$$

$$= \frac{(x-6)^2}{x(x+5)}, \quad x \neq -5, -4, 0, 5, 6. \quad \left[ \text{state all the restrictions after simplified answer.} \right]$$

**Practice: Multiplying and Dividing Rational Expressions**

1. Simplify and state the restrictions on the variables.

a)  $\frac{3x}{8} \times \frac{2y}{9}$

$$= \frac{(\cancel{3})(\cancel{2})(xy)}{(\cancel{2})(4)(\cancel{3})(3)}$$

$$= \frac{xy}{12}$$

b)  $\frac{9x^2y}{4y} \times \frac{12xy}{3x^2y}$

$$= \frac{3 \cdot 3 \cdot 4 \cdot 3 \cdot x^2y^2}{4 \cdot 3 \cdot x^2y^2}$$

$$= 9x, \quad x \neq 0, y \neq 0$$

c)  $\frac{14pq}{5q^2} \times \frac{10q^3}{21pq^2}$

$$= \frac{4}{3}, \quad p, q \neq 0$$

d)  $\frac{-8x^2}{10y} \times \frac{5y^3}{-2x^2}$

$$= \frac{2x^2y^3}{x^2y} = 2y^2$$

$x \neq 0, y \neq 0$

2. Simplify and state the restrictions.

a)  $\frac{8a}{9} \div \frac{2b}{3}$

$$= \frac{8a}{9} \times \frac{3}{2b}$$

$$= \frac{4a}{3b}, \quad b \neq 0$$

b)  $\frac{-56a^2b^2}{24a^3b} \div \frac{16ab^3}{18b}$

$$= \frac{-8 \cdot 7 a^2 b^2 \cdot 3}{8 \cdot 3 \cdot a^3 b \cdot 8 \cdot 2 b^3}$$

$$= \frac{-21}{8a^2b}, \quad a \neq 0, b \neq 0$$

c)  $\frac{-5a^3}{3b^2} \div \frac{-10a^2b}{3b^2}$

$$= \frac{(5a^3)(3b^2)}{(3b^2)(-10a^2b)}$$

$$= \frac{a}{2b}, \quad a, b \neq 0$$

3. Express each product in lowest terms.

a)  $\frac{12y}{3y-9} \times \frac{4y-12}{6y^2}$

$$= \frac{(\cancel{12}y)(4)(y-3)}{3(y-3)(6)(y^2)}$$

$$= \frac{8}{3y}$$

b)  $\frac{6p-12}{5p+5} \times \frac{2p+2}{3p-6}$

$$= \frac{6(p-2)(2)(p+1)}{5(p+1)(3)(p-2)}$$

$$= \frac{4}{5}, \quad p \neq -1, 2$$

c)  $\frac{x^2-y^2}{x^2-16} \times \frac{x-4}{x-y}$

$$= \frac{(x-y)(x+y)(x-4)}{(x-4)(x+4)(x-y)}$$

$$= \frac{x+y}{x+4}, \quad x \neq y, \neq 4$$

4. Simplify each quotient.

a)  $\frac{k+1}{k^2-1} \div \frac{k}{k-1}$

$$= \frac{k+1}{(k-1)(k+1)} \times \frac{k-1}{k}$$

$$= \frac{1}{k}, k \neq \pm 1, 0.$$

b)  $\frac{x^2-4}{x+3} \div \frac{x-2}{x^2-9}$

$$= \frac{(x-2)(x+2)(x-3)(x+3)}{(x+3)(x-2)}$$

$$= (x+2)(x-3), x \neq \pm 3, 2$$

c)  $\frac{1-3q}{2q+1} \div \frac{1-9q^2}{4q^2-1}$

$$= \frac{(1-3q)}{(2q+1)} \times \frac{(2q-1)(2q+1)}{(1-3q)(1+3q)}$$

$$= \frac{2q-1}{1+3q}, q \neq \frac{-1}{2}, \frac{1}{3}$$

5. Explain why there may be more restrictions when you divide rational expressions than when you multiply the same two expressions. Use an example to illustrate your explanation.

$\div \rightarrow \times$  by reciprocating the second rational expression which results in more restrictions.

eg.  $\frac{x}{x+1} \div \frac{x+5}{x-2}$

$$\Rightarrow \frac{x}{x+1} \times \frac{x-2}{x+5}$$

current restrictions  
 $x \neq -1, 2$

one more restriction

6. Simplify.

a)  $\frac{x^2+2x+1}{x} \times \frac{x^2-x}{x^2-1}$

$$= \frac{(x+1)(x+1)(x)(x-1)}{x(x-1)(x+1)}$$

$$= x+1, x \neq \pm 1, 0.$$

b)  $\frac{2x^2-x-3}{x^2-1} \times \frac{x^2+x-2}{2x^2+x-6}$

$$= \frac{(2x-3)(x+1)(x+2)(x-1)}{(x-1)(x+1)(2x-3)(x+2)}$$

$$= 1, x \neq \pm 1, \frac{3}{2}, -2$$

c)  $\frac{x^2+5x+6}{x^2-4} \times \frac{x^2-6x+8}{x^2-x-12}$

$$= \frac{(x+3)(x+2)(x-4)(x-2)}{(x-2)(x+2)(x-4)(x+3)}$$

$$= 1, x \neq -3, \pm 2, 4.$$

7. Simplify.

a)  $\frac{x^2-1}{x^2+6x+9} \div \frac{x^2+2x-3}{x^2+6x+9}$

$$= \frac{(x-1)(x+1)(x+3)(x+3)}{(x+3)(x+3)(x+3)(x-1)}$$

$$= \frac{x+1}{x+3}, x \neq -3, 1$$

b)  $(4x^2-9y^2) \div \frac{3xy+2x^2}{2}$

$$= \frac{(2x-3y)(2x+3y)(2)}{x(3y+2x)}$$

$$= \frac{2(2x-3y)}{x}, x \neq 0, -\frac{3}{2}y$$

c)  $\frac{a^2+10a+25}{a^2+2a-15} \times \frac{a^2-5a+6}{a^2+3a-10}$

$$= \frac{(a+5)(a+5)(a-3)(a-2)}{(a+5)(a-3)(a+5)(a+2)}$$

$$= 1, a \neq -5, 2, 3.$$

8. An estimate of the cost, in billions of dollars, to keep the atmosphere in Canada  $x$  percent free of a chemical toxin is given by

$$C(x) = \frac{x-2}{100x-x^2} \div \frac{x^2+18x-40}{5x^2+100x}. \text{ Find the cost of keeping the atmosphere 90\% free of the chemical toxin.}$$

Simplify

$$\frac{\cancel{x-2}}{x(100-x)} \cdot \frac{(5x)(\cancel{x+20})}{(x+20)(\cancel{x+2})}$$

$$= \frac{5}{100-x}$$

$$C(90) = \frac{5}{100-90}$$

$$= \frac{5}{10} = 0.5$$

$\therefore$  0.5 billion dollars.

9. Charlene could not evaluate the expression  $\frac{15x^2+7x-2}{2x^2-x-15} \div \frac{6x^2-11x-10}{4x^2-25}$  for  $x=2.5$ . Simplify the expression and explain why.

$$= \frac{(5x-1)(3x+2)(\cancel{2x-5})(\cancel{2x+5})}{(\cancel{2x+5})(x-3)(3x+2)(\cancel{2x-5})}$$

$$= \frac{5x-1}{x-3}, \quad x \neq \frac{-5}{2}, 3, \frac{-2}{3}, \frac{5}{2}$$

$$x=2.5: \quad \frac{5(2.5)-1}{2.5-3} = \frac{12.5-1}{-0.5} = \frac{11.5}{-0.5} = -23$$

$$10. \text{ Simplify } \frac{\frac{m^2-mn}{6m^2+11mn+3n^2} \div \frac{m^2-n^2}{2m^2-mn-6n^2}}{\frac{4m^2-7mn-2n^2}{3m^2+7mn+2n^2}} = \left( \frac{(m)(m-n)(2m+3n)(m-2n)}{(3m+n)(2m+n)(m-n)(m+n)} \right) \div \left( \frac{(4m+n)(m-2n)}{(3m+n)(m+2n)} \right)$$

$$= \frac{(m)(m-n)(2m+3n)(\cancel{m-2n})(3m+n)(m+2n)}{(3m+n)(2m+3n)(\cancel{m-n})(m+n)(4m+n)(\cancel{m-2n})}$$

$$= \frac{m(m+2n)}{(m+n)(4m+n)}, \quad m \neq -\frac{n}{3}, \frac{-3}{2}n, \frac{1}{2}n, 2n, \frac{-1}{4}n, \frac{-1}{3}n, -2n$$