Day3-MCR3U

RATIONAL EXPRESSIONS

If x and y are two integers, then the quotient $\frac{x}{y}$ is a rational number with the restriction $y \neq 0$

If P and Q are polynomials, then the quotient $\frac{P}{Q}$ is a rational expression with the restriction $Q \neq 0$

Examples of rational numbers: a) $\frac{3}{5}$ b) $-8 = \frac{-8}{1}$ c) $0.13 = \frac{13}{100}$

Examples of rational expressions: a) $\frac{3+x}{5-x}$ b) $\frac{x^2-2x}{x^2}$ c) $\frac{3x}{9-x^2}$

d) $\frac{1}{xy}$ e) $\frac{3x}{x^2+4}$ f) $\frac{-2x^2}{x^2+4x-5}$

RESTRICTIONS

Because the denominator cannot equal 0, we must restrict values of x so that the denominator does not equal 0. You may need to factor the denominator to find the restrictions.

State the restrictions for each of the following:

a) $\frac{3+x}{5-x}$ b) $\frac{x^2-2x}{x^2}$ c) $\frac{3x}{9-x^2}$ d) $\frac{1}{xy}$ e) $\frac{3x}{x^2+4}$ f) $\frac{-2x^2}{x^2+4x-5}$

a) $5-x \neq 0$

$x \neq 5$

b) $x \neq 0$

c) $9-x^2 \neq 0$

$x^2 \neq 9$

$x \neq \pm 3$

or $(3-x)(3+x) \neq 0$

$x \neq \pm 3$

d) $x \neq 0$

$y \neq 0$

e) $x^2+4 \neq 0$

No restriction

$x^2+4 > 0$

f) $(x-1)(x+5)$

$x \neq 1, -5$

SIMPLIFYING RATIONAL EXPRESSIONS

To simplify rational expressions we use the same techniques as with simplifying rational numbers:

Example 1:	Example 2:	Example 3:
$\frac{30}{42}$	$\frac{x^2-2x}{x^2}$	$\frac{x^2-9}{x^2-2x-15}$
$=\frac{(2)(3)(5)}{(2)(3)(7)}$	$=\frac{x(x-2)}{(x)(x)}$	$=\frac{(x-3)(x+3)}{(x-5)(x+3)}$
$=\frac{5}{7}$	$=\frac{x-2}{x}$	$=\frac{x-3}{x-5}$
	State restriction: $x \neq 0$	State restrictions: $x \neq -3, +5$

Simplify the rational expressions and state any restrictions on the variables.

1) $\frac{30x^4y^3}{-6x^7y}$

$= -5x^{-3}y^2$

$= \frac{-5y^2}{x^3}$

$x \neq 0, y \neq 0$

2) $\frac{10x^4-8x^2+4x}{2x^2}$

$= \frac{2x(5x^3-4x+2)}{2x^2}$

$= \frac{(5x^3-4x+2)}{x}, \quad x \neq 0.$

3) $\frac{2x^2+3x-2}{x^2-4}$

$= \frac{2x^2+4x-x-2}{(x-2)(x+2)}$

$= \frac{(2x-1)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}} = \frac{2x-1}{x-2}$

$x \neq \pm 2$

4) $\frac{x-7}{14-2x}$

$= \frac{x-7}{-2(x-7)}$

$= -\frac{1}{2}, \quad x \neq 7.$

Note: We can **NOT** cancel term by term. We can **ONLY** cancel factors. For example, we can not cancel 'y' in the expression: $\frac{y+p}{y}$. However, if 'p' and 'y' were multiplied in the numerator, then we can cancel 'y'.

can not do this $\frac{\cancel{5}+10}{\cancel{5}}$

$\frac{(\cancel{5})(10)}{\cancel{5}} = 10 \rightarrow$ can do this.

Practice: Simplifying Rational Expressions

1. Simplify and state the restrictions on the variables.

a) $\dfrac{2b+8}{5b+20}$ b) $\dfrac{m^2-4m}{3m^2-12m}$ c) $\dfrac{x^2-4}{x^2-5x+6}$

$= \dfrac{2(b+4)}{5(b+4)}$ $= \dfrac{m(m-4)}{3m(m-4)}$ $= \dfrac{\cancel{(x-2)}(x+2)}{(x-3)\cancel{(x-2)}}$

$= \dfrac{2}{5}, b \neq -4$ $= \dfrac{1}{3}, m \neq 0, 4$ $= \dfrac{x+2}{x-3}, x \neq 2, 3$

2. Simplify and state the restrictions.

a) $\dfrac{36x^2y}{-16xy^2} = \dfrac{9 \cdot \cancel{4} \cdot \cancel{x} \cdot x \cdot \cancel{y}}{-4 \cdot \cancel{4} \cdot \cancel{x} \cdot \cancel{y} \cdot y}$

$= \dfrac{9 \cdot x}{-4y}, x \neq 0$
$y \neq 0$

b) $\dfrac{x^2-10xy+25y^2}{x^2-25y^2}$

$= \dfrac{\cancel{(x-5y)}(x-5y)}{\cancel{(x-5y)}(x+5y)}$

$= \dfrac{x-5y}{x+5y}, x \neq \pm 5y.$

3. Simplify and state the restrictions.

a) $\dfrac{6m^2-2m-4}{4m^2-4}$ b) $\dfrac{9x^2-1}{9x^3+6x^2+x}$

$= \dfrac{2(3m^2-m-2)}{4(m^2-1)}$ $= \dfrac{(3x-1)(3x+1)}{x(9x^2+6x+1)}$ Perfect square trinomial

$= \dfrac{2(3m+2)\cancel{(m-1)}}{4\cancel{(m-1)}(m+1)}$ $= \dfrac{(3x-1)\cancel{(3x+1)}}{x(3x+1)\cancel{(3x+1)}}$

$= \dfrac{3m+2}{2(m+1)}, m \neq -1, 1$ $= \dfrac{3x-1}{x(3x+1)}, x \neq 0, -\dfrac{1}{3}$

Solutions:

1. a) $\dfrac{2}{5}, b \neq -4$ b) $\dfrac{1}{3}, m \neq 0,4$ c) $\dfrac{x+2}{x-3}, x \neq 2,3$

2. a) $-\dfrac{9x}{4y}, x \neq 0, y \neq 0$ b) $\dfrac{x-5y}{x+5y}, x \neq \pm 5y$ 3. a) $\dfrac{3m+2}{2(m+1)}, m \neq \pm 1$ b)

$\dfrac{3x-1}{x(3x+1)}, x \neq 0, -\dfrac{1}{3}$

4. What does the word *restrictions* mean in connection with a rational expression, and why must you state the restrictions? Explain using an example.

See the answer below.

5. a) What is the relation between the expressions x – y and y – x?

$x-y = -(y-x)$

b) Simplify $\dfrac{x^2+xy-2y^2}{y^2-x^2}$ and state the restrictions on the variable.

$$= \frac{(x+2y)(x-y)}{(y-x)(y+x)} = \frac{(x+2y)\cancel{(x-y)}}{-\cancel{(x-y)}(y+x)} = -\frac{x+2y}{x+y},\ x \neq \pm y$$

6. Simplify and state the restrictions.

a) $\dfrac{4a^2-b^2}{b-2a}$

$$= \frac{\cancel{(2a-b)}(2a+b)}{-\cancel{(2a-b)}} = -(2a+b),\ b \neq 2a$$

b) $\dfrac{x^2-2x+1}{1-x}$

$$= \frac{(x-1)\cancel{(x-1)}}{-\cancel{(x-1)}} = -(x-1),\ x \neq 1$$

c) $\dfrac{k^2-10k+25}{25-k^2}$

$$= \frac{(k-5)(k-5)}{(5-k)(5+k)} = \frac{\cancel{(k-5)}(k-5)}{-\cancel{(k-5)}(k+5)} = -\frac{k-5}{k+5},\ k \neq \pm 5$$

Homework: p. 113 #1-3, 4bdf, 5, 6ce, 10, 13

Solutions:

4. Restrictions are limitations set on the value of a variable. In a rational expression, $\frac{1}{0}$ is undefined, so the variable can not equal a value that results in the denominator being equal to 0. Restrictions must be stated to avoid undefined values. Ex $\frac{1}{x+7}$ is only defined if $x \neq -7$.

5. a) x – y = -(y – x) b) $-\dfrac{x+2y}{x+y}, x \neq \pm y$ 6. a) –(2a + b), $b \neq 2a$ b) –(x – 1), $x \neq 1$

c) $-\dfrac{k-5}{k+5}, k \neq \pm 5$