Evaluate each of the following.

a. $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\frac{1}{2}+\frac{1}{2}} = 9$

b. $\sqrt{9} \times \sqrt{9} = (\sqrt{9})^2 = 9$

Therefore, $9^{\frac{1}{2}} = \sqrt{9}$

In general $a^{\frac{1}{2}} = \sqrt{a}$, $a \geq 0$

c. $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 8$

d. $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = (\sqrt[3]{8})^3 = 8$

Therefore: $8^{\frac{1}{3}} = \sqrt[3]{8}$

In general: $b^{\frac{1}{3}} = \sqrt[3]{b}$

Exponents in the form $\frac{1}{n}$

$x^{\frac{1}{n}} = \sqrt[n]{x}$ where n is a natural number. (Read the n^{th} root of x)

Examples: Write each of the following in radical form. Evaluate, if possible.

a. $64^{\frac{1}{2}}$
$= \sqrt{64}$
$= 8$

b. $27^{\frac{1}{3}}$
$= \sqrt[3]{27}$
$= 3$

c. $1024^{\frac{1}{5}}$
$= \sqrt[5]{1024}$
$= 4$

d. $16^{\frac{1}{4}}$
$= \sqrt[4]{16}$
$= 2$

e. $9^{-\frac{1}{2}}$
$= \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

f. $(-64)^{\frac{1}{3}}$
$= \sqrt[3]{-64}$
$= -4$

g. $(-243)^{\frac{1}{5}}$
$= \sqrt[5]{-243}$
$= -3$

h. $(-625)^{\frac{1}{4}}$
$= \sqrt[4]{-625}$
Not Possible
(Even root of a negative not defined)

i. $(-81)^{\frac{1}{2}}$
$= \sqrt{-81}$
NP.

Take Note:

Given $\sqrt[n]{x}$, if n is an even number, then $x \geq 0$ for the n^{th} root to be real.
if n is an odd number, then x can be any real number.

Use the exponent laws to express $x^{\frac{2}{3}}$ in two ways. **Recall:** Power law $(a^m)^n = a^{mn}$

a. $x^{\frac{2}{3}} = (x^2)^{\frac{1}{3}} = \sqrt[3]{x^2}$ **b.** $x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = (\sqrt[3]{x})^2$

Exponents in the form $\frac{m}{n}$:

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$ where m and n are natural numbers.

Examples: Write each of the following in radical form. Evaluate, if possible.

a. $8^{\frac{2}{3}} = (\sqrt[3]{8})^2$
$= 2^2$
$= 4$

b. $-25^{\frac{5}{2}}$
$= -(\sqrt{25})^5$
$= -(5)^5$
$= -3125$

c. $256^{-\frac{3}{4}}$
$= \frac{1}{256^{3/4}} = \frac{1}{(\sqrt[4]{256})^3}$
$= \frac{1}{4^3} = \frac{1}{64}$

d. $64^{-1.5}$
$= 64^{-\frac{3}{2}}$
$= \frac{1}{64^{3/2}} = \frac{1}{(\sqrt{64})^3}$
$= \frac{1}{8^3} = 512$

e. $(-27)^{-\frac{2}{3}}$
$= \frac{1}{(-27)^{\frac{2}{3}}}$
$= \frac{1}{(\sqrt[3]{-27})^2}$
$= \frac{1}{(-3)^2} = \frac{1}{9}$

f. $-27^{-\frac{2}{3}}$
$= -\frac{1}{27^{\frac{2}{3}}}$
$= -\frac{1}{(\sqrt[3]{27})^2}$
$= -\frac{1}{9}$

Practice: Evaluate. (Show at least one middle step)

a. $(\frac{4}{9})^{\frac{3}{2}} = (\sqrt{\frac{4}{9}})^3$
$= (\frac{2}{3})^3 = \frac{8}{27}$

b. $(\frac{27}{64})^{\frac{2}{3}} = (\sqrt[3]{\frac{27}{64}})^2$
$= (\frac{3}{4})^2 = \frac{9}{16}$

c. $(\frac{-32}{243})^{\frac{2}{5}} = (\sqrt[5]{\frac{-32}{243}})^2$
$= (-\frac{2}{3})^2 = \frac{4}{9}$

d. $(\frac{1}{16})^{-\frac{1}{2}} = (\frac{16}{1})^{\frac{1}{2}}$
$= \sqrt{\frac{16}{1}} = 4$

e. $(\frac{64}{125})^{-\frac{2}{3}}$
$= (\frac{125}{64})^{\frac{2}{3}} = (\sqrt[3]{\frac{125}{64}})^2 = (\frac{5}{4})^2$
$= 25/16$

f. $(\frac{-32}{243})^{-\frac{2}{5}}$
$= (\frac{243}{-32})^{\frac{2}{5}}$
$= (\sqrt[5]{\frac{243}{-32}})^2 = (\frac{3}{-2})^2$
$= 9/4$