Day2-MCR3U

Name:_____
Date:_____

Factoring - Note

There are 4 things to look for when factoring:

1) Common Factor
2) Difference of Squares
3) Simple Trinomial
4) Complex Trinomial

1) Common Factor

- find the largest number and variable combination that go into each term
- divide each term by the common factor

Ex. Factor $6x^3 - 12x^2 + 24x$
Solution: $= 6x(x^2 - 2x + 4)$

2) Difference of Squares

- look for $a^2 - b^2$, since the two factors are $(a + b)$ and $(a - b)$

Ex. Factor $36x^2 - 49y^2$
Solution: $= (6x - 7y)(6x + 7y)$

3) Simple Trinomial

- look for $x^2 + bx + c$, i.e. the coefficient of the x^2 term is 1
- find 2 numbers, m and n, that multiply to c and add to b
- the two factors are $(x + m)$ and $(x + n)$

Ex. Factor $x^2 + 5x + 6$
two numbers that multiply to 6 and add to 5 are: 3 and 2
Solution: $x^2 + 5x + 6 = (x + 3)(x + 2)$

Ex. Factor $x^2 - 6x - 7$
two numbers that multiply to -7 and add to -6 are: 1 and -7
Solution: $x^2 - 6x - 7 = (x - 7)(x + 1)$

4) Complex Trinomial

- look for $ax^2 + bx + c$, i.e. the coefficient of the x^2 term is not 1
- find 2 numbers, m and n, that multiply to a x c and add to b
- decompose (break) the middle term into 2 pieces: mx and nx
- common factor the first 2 terms and the last 2 terms
- write the solution as the product of the 2 factors

Ex. Factor $6x^2 - x - 12$
two numbers that multiply to -72 and add to -1 are: -9 and +8
Solution: $6x^2 - x - 12 = 6x^2 - 9x + 8x - 12$

$= 3x(2x - 3) + 4(2x - 3)$ * note: 1 factor should be the same!!

$= (3x + 4)(2x - 3)$

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Factoring Review Continued

1. $144x^2 - 25y^2$

$= (12x - 5y)(12x + 5y)$

2. $4x^2 + 25$

D.N.F

(sum of squares)

3. $8p^3 + 4p^2 - 2p - 1$

$= 4p^2(2p+1) - (2p+1)$

$= (4p^2 - 1)(2p+1)$

$= (2p-1)(2p+1)(2p+1)$

$= (2p+1)^2(2p-1)$

4. $(7x + 4y)^2 - (8x - 3y)^2$

$= (7x+4y+8x-3y)(7x+4y-(8x-3y))$

$= (15x + y)(-x + 7y)$

5. $2x^2 + 4x + 1$

$=$ D.N.F

6. $9p^2 - 30p + 25$

$= 9p^2 - 15p - 15p + 25$

$= 3p(3p-5) - 5(3p-5)$

$= (3p-5)^2$

7. $12a^2 + 12a + 3$

$= 3(4a^2 + 4a + 1)$

$= 3(4a^2 + 2a + 2a + 1)$

$= 3(2a(2a+1) + (2a+1))$

$= 3(2a+1)^2$

8. $-x^2 - 3x + 40$

$= -(x^2 + 3x - 40)$

$= -(x+8)(x-5)$

9. $25x^2 - 20xy + 4y - 49$

$= (25x^2 - 49) + (-20xy + 28y)$

$= (5x-7)(5x+7) - 4y(5x-7)$

$= (5x-7)(5x+7-4y)$

10. $16x^2(2x-1) - 49(2x-1)$

$= (16x^2 - 49)(2x-1)$

$= (4x-7)(4x+7)(2x-1)$

11. $16x^4 - 81y^4$

$= (4x^2 - 9y^2)(4x^2 + 9y^2)$

$= (2x-3y)(2x+3y)(4x^2+9y^2)$

12. $2x^2 + 3x - 2$

$= 2x^2 + 4x - x - 2$

$= 2x(x+2) - (x+2)$

$= (2x-1)(x+2)$

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Solving Equations – Note

Solving Equations

- look for $ax^2 + bx + c = 0$, i.e. the right hand side = 0
- if you have an x^2 term, then rearrange the equation so that the right hand side is 0

- factor, if possible, then set each factor = 0 to find the value of x that solves the equation

- if factoring is not possible, use the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex. Solve $x^2 + 3x + 2 = 0$
Solution: $(x + 1)(x + 2) = 0$
If $x + 1 = 0$, then $x = -1$
If $x + 2 = 0$, then $x = -2$
Therefore the solution is $x = -1$ or $x = -2$.

Ex. Solve $2x^2 - 7x - 4 = 0$
Solution: $2x^2 - 8x + x - 4 = 0$
$2x(x - 4) + 1(x - 4) = 0$
$(x - 4)(2x + 1) = 0$
If $x - 4 = 0$, then $x = 4$
If $2x + 1 = 0$, then $x = -\frac{1}{2}$
Therefore the solution is $x = 4$ or $x = -\frac{1}{2}$.

Ex. Solve $2x^2 + 3x + 4 = 5 - 2x$, correct to 2 decimal places
Rearrange so RHS = 0: $2x^2 + 5x - 1 = 0$ (cannot be factored, so use quadratic formula)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 + 8}}{4}$$

Exact answer: $x = \dfrac{-5 \pm \sqrt{33}}{4}$ *exact answer: stop here

Approx answer: $x = \dfrac{-5 + 5.745}{4}$ or $x = \dfrac{-5 - 5.745}{4}$ *carry at least 1 extra decimal place until the final answer

$x = 0.19$ or $x = -2.69$
Therefore the solution is $x = 0.19$ or $x = -2.69$.

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Factoring and Solving Quadratic Equations

1. Factor completely.

a) $-3xyz - 21x^2yz$

$= -3xyz(1+7x)$

b) $25m^2 - 49$

$= (5m-7)(5m+7)$

c) $a^2 + 5a + 6$

$= (a+3)(a+2)$

d) $6x^2 - 7x - 5$

$= 6x^2 - 10x + 3x - 5$

$= 2x(3x-5) + (3x-5)$

$= (2x+1)(3x-5)$

e) $64x^2 - 9y^2$

$= (8x-3y)(8x+3y)$

f) $14m^2n + 7mn - 21n^2$

$= 7(2m^2 + mn - 3n^2)$

$= 7(2m+3n)(m-n)$

g) $3m^2 - 5mn - 2n^2$

$= 3m^2 - 6mn + mn - 2n^2$

$= 3m(m-2n) + n(m-2n)$

$= (3m+n)(m-2n)$

h) $x^2 - 6xy - 16y^2$

$= x^2 - 8xy + 2xy - 16y^2$

$= x(x-8y) + 2y(x-8y)$

$= (x+2y)(x-8y)$

2. Factor completely.

a) $4s^3 + 12s^2 - 16s$

$= 4s(s^2 + 3s - 4)$

$= 4s(s+4)(s-1)$

b) $36x^3 - 84x^2 + 24x$

$= 12x(3x^2 - 7x + 2)$

$= 12x(3x^2 - 6x - x + 2)$

$= 12x(3x(x-2) - (x-2)) = 12x(3x-1)(x-2)$

c) $4x^2 - 26x - 14$

$= 2(2x^2 - 13x - 7)$

$= 2(2x^2 - 14x + x - 7)$

$= 2(2x(x-7) + (x-7))$

$= 2(2x+1)(x-7)$

d) $3a^2 + 18a + 27$

$= 3(a^2 + 6a + 9)$

$= 3(a+3)(a+3)$

$= 3(a+3)^2$

e) $5t^2 - 5t - 30$

$= 5(t^2 - t - 6)$

$= 5(t-3)(t+2)$

f) $18x^2 + 15x - 12$

$= 3(6x^2 + 5x - 4)$

$= 3(6x^2 + 8x - 3x - 4)$

$= 3(2x(3x+4) - (3x+4))$

$= 3(2x-1)(3x+4)$

Solutions

1. a) $-3xyz(1 + 7x)$ b) $(5m + 7)(5m - 7)$ c) $(a + 3)(a + 2)$ d) $(3x - 5)(2x + 1)$
   e) $(8x + 3y)(8x - 3y)$ f) $7n(2m^2 + m - 3n)$ g) $(m - 2n)(3m + n)$ h) $(x - 8)(x + 2)$
2. a) $4s(s + 4)(s - 1)$ b) $12x(x - 2)(3x - 1)$ c) $2(x - 7)(2x + 1)$ d) $3(a + 3)^2$
   e) $5(t - 3)(t + 2)$ f) $3(2x - 1)(3x + 4)$

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3. Solve by factoring.

a) $x^2 - 4x - 12 = 0$

$(x-6)(x+2)=0$

$x=6$ or $x=-2$

b) $x^2 - 8x + 15 = 0$

$(x-5)(x-3)=0$

$x=5$ or $x=3$.

c) $5x = 6x^2 + 1$

$6x^2 - 5x + 1 = 0$

$6x^2 - 3x - 2x + 1 = 0$

$3x(2x-1) - (2x-1) = 0$

$(3x-1)(2x-1)=0$

$x=\frac{1}{3}$ or $x=\frac{1}{2}$

d) $50x^2 - 162 = 0$

$2(25x^2 - 81) = 0$

$2(5x-9)(5x+9)=0$

$x = \pm\frac{9}{5}$

4. Solve using the quadratic formula – exact roots.

a) $x^2 - 3x - 5 = 0$

$x = \frac{3 \pm \sqrt{9-4(1)(-5)}}{2}$

$= \frac{3 \pm \sqrt{29}}{2}$

b) $4y^2 + 7y - 9 = 0$

$y = \frac{-7 \pm \sqrt{49-4(4)(-9)}}{8}$

$= \frac{-7 \pm \sqrt{193}}{8}$

5. Solve using the quadratic formula – approximate roots, rounded to 2 decimal places.

a) $2j^2 + 5j - 4 = 0$

$j = \frac{-5 \pm \sqrt{25-4(2)(-4)}}{4}$

$= \frac{-5 \pm \sqrt{57}}{4}$

$j_1 = \frac{-5+\sqrt{57}}{4} = 0.64$

$j_2 = \frac{-5-\sqrt{57}}{4} = -3.14$

b) $3g^2 + 2 = 11g$

Homework: p. 102 #1-6 (p. 107 #6-10)

Solutions

3. a) x = 6, x = -2 b) x = 5, x = 3 c) $x=\frac{1}{2}, x=\frac{1}{3}$ d) $x=\frac{9}{5}, x=-\frac{9}{5}$

4. a) $x=\frac{3\pm\sqrt{29}}{2}$ b) $y=\frac{-7\pm\sqrt{193}}{8}$

5. a) j = 0.64, j = -3.14 b) g = 3.47, g = 0.19