5.1: Primary and Reciprocal Ratios

Right Angle Triangles

From a given angle, we can name the sides of a right angle triangle:

Adjacent is the side that touches the angle. **Opposite** is the side that does not touch the angle.

***Note:** The opposite and adjacent sides depend on the angle! If we look at the same triangle but a different angle, the opposite and adjacent sides will be different, but the Hypotenuse always stays the same!

Primary Trigonometric Ratios

In a **right triangle**, the angles and the lengths of sides are related by certain ratios:

The three primary trigonometric ratios are:

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

θ = is the reference angles (acute angles)

Hint: Take the first letter of each word.

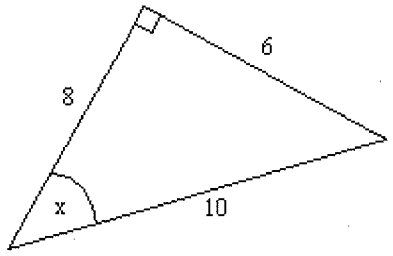
S=OH C=AH T=OA

"**Solve**" a triangle means find the missing angles and sides. Use your math tools:

Trig ratios SOH CAH TOA, Pythagorean Theorem $a^2+b^2=c^2$

Sum of Angles in a Triangle is 180°

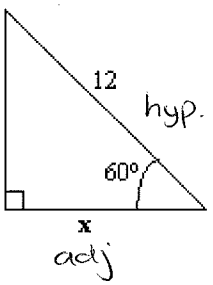
Example 1: For the triangle below, find x



$x = \sin^{-1}(3/5) \doteq 37^\circ$

a) $\sin x = \frac{6}{10} = \frac{3}{5}$ b) $\cos x = \frac{8}{10} = \frac{4}{5}$ c) $\tan x = \frac{6}{4} = \frac{3}{2}$

Example 2: Find the side length



$\cos 60^\circ = \frac{x}{12}$

$12(\cos 60^\circ) = x$

$x = 6$

Primary Trig ratios

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

Reciprocal Trig Ratios

$\text{cosecant}\,\theta = \frac{1}{\sin\theta}$ $\text{secant}\,\theta = \frac{1}{\cos\theta}$ $\text{cotangent}\,\theta = \frac{1}{\tan\theta}$

Calculators don't have reciprocal trig buttons, so if you to evaluate $\sec 20° = \frac{1}{\cos 20°}$

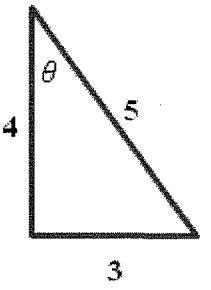
$\doteq 1.0642$

Example 3:

a) Determine csc θ, sec θ and cot θ

$\csc\theta = \frac{1}{\sin\theta} = \frac{H}{O} = \frac{5}{3}$ $\sec\theta = \frac{1}{\cos\theta} = \frac{H}{A} = \frac{5}{4}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{A}{O} = \frac{4}{3}$



b) Calculate θ to the nearest degrees.

$\sin\theta = \frac{3}{5}$

$\theta = \sin^{-1}(3/5) \doteq 37°$

Example 4: Solve the triangle. (ROUNDING: Angles nearest degree, Sides one decimal place)

$\sin P = \frac{13.2}{28.4}$

$\angle P \doteq 28°$

$\angle R = 180° - 90° - 28$

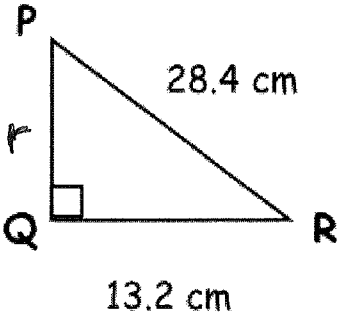
$= 62°$

$\sin R = \frac{r}{28.4}$

$\sin 62 = \frac{r}{28.4}$

$r = (28.4)(\sin 62)$

$= 25.1$ cm.



(Answers may differ in decimals).

Day1-MCR3U

Reciprocal Trig Ratios

1. Find csc B.

a) sin B = 0.9781

$\csc B = \frac{1}{0.9781}$

$= 1.0224$

b) sin B = 0.4067

$\csc B = \frac{1}{\sin B}$

$= \frac{1}{0.4067}$

$= 2.4588$

c) B = 37°

$\csc B = \frac{1}{\sin B}$

$= \frac{1}{\sin 37°}$

$= 1.6616$

d) B = 103°

$\csc B = \frac{1}{\sin 103}$

$= 1.0263$

2. Find sec C.

a) cos C = 0. 4848

$\sec C = \frac{1}{\cos C} = \frac{1}{0.4848}$

$= 2.0627$

b) cos C = 0.9272

$\sec C = \frac{1}{0.9272}$

$= 1.0785$

c) C = 81°

$\sec C = \frac{1}{\cos C}$

$= \frac{1}{\cos 81} = 6.3924$

d) C = 112°

$\sec C = \frac{1}{\cos C}$

$= \frac{1}{\cos 112}$

$= -2.6695$

3. Find cot D.

a) tan D = 2.0503

$\cot D = \frac{1}{\tan D} = \frac{1}{2.0503}$

$= 0.4877$

b) tan D = 0.2493

$\cot D = \frac{1}{\tan D} = \frac{1}{0.2493}$

$= 4.0112$

c) D = 42°

$\cot D = \frac{1}{\tan 42}$

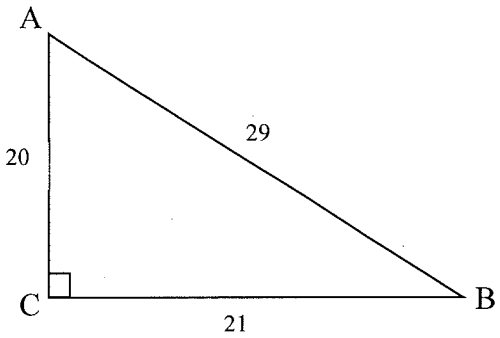
$= 1.1106$

d) D = 121°

$\cot D = \frac{1}{\tan 121}$

$= -0.6009$

4. Find the desired ratios.



a) $\sin A = \frac{21}{29}$

b) $\cos B = \frac{21}{29}$

c) $\csc A = \frac{29}{21}$

d) $\sec A = \frac{29}{20}$

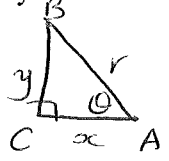
e) $\csc B = \frac{29}{20}$

f) $\cot B = \frac{21}{20}$

5. From #4, what do you notice about sin A and cos B? What do you notice about sec A and csc B? Will this always be true? Why or why not?

$\sin A = \cos B$

$\frac{y}{r} \quad \frac{y}{r}$



$\sec A = \csc B$

$\frac{r}{y} \quad \frac{r}{y}$

Solutions:

1. a) 1.0224 b) 2.4588 c) 1.6616 d) 1.0263
2. a) 2.0627 b) 1.0785 c) 6.3925 d) –2.6695
3. a) 0.4877 b) 4.0112 c) 1.1106 d) –0.6009
4. a) 21/29 b) 21/29 c) 29/21 d) 29/20 e) 29/20 f) 21/20
5. they are the same, same, yes

Homework: Handout, p. 280 #1,2,4,5iv,6,8b,9,10, 14,15