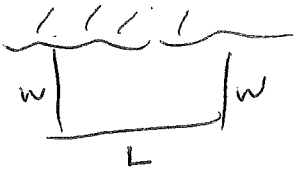
Date: _____

Completing the Square to Solve Word Problems

1. A rectangular field is to be enclosed on one side by a barn and on the other three sides by 400m of fencing. Determine the dimensions of the field if the area is to be a maximum.



$2w + L = 400$

$L = 400 - 2w$.

$A = L \cdot w$

$= (400 - 2w)(w)$

$= 400w - 2w^2$

$= -2w^2 + 400w$

$= -2(w^2 - 200w + 10000) - 10000$

$= -2(w - 100)^2 + 20000$

$\therefore w = 100\text{m}$

$L = 400 - 2w = 200\text{m}$

2. Find two integers whose difference is 12 and whose product is a minimum.

Let x and $x-12$ represent the integers

$P = (x)(x-12)$

$= x^2 - 12x$

$= x^2 - 12x + 36 - 36$

$= (x-6)^2 - 36$

$x = 6 \quad P = -36.$

$\therefore$ 6 and -6 are the integers.

↳ $x - 12 = 6 - 12 = -6$

MPM2D1 Date: _____

Day 12: Completing the Square to Solve Word Problems

Unit 5: Quadratic Relations

Revenue Problems:

3. A motel has 30 rooms. The owner decides to raise the price of each room from the current price of $40. Every $2 increase in price results in one less room rented out. What price of each room would maximize revenue?

Rooms	Price	R
30	40	1200
29	42	1218
28	44	1232
27	46	1242
26	48	1248

$R = (30-x)(40+2x)$

$= 1200 + 20x - 2x^2$

$= -2x^2 + 20x + 1200$

$= -2(x^2 - 10x) + 1200$

$= -2(x^2 - 10x + 25 - 25) + 1200$

$= -2(x-5)^2 + 1250$

$\therefore (5, 1250) \quad \therefore P = \$50$ will maximize the R.

4. Calculators are sold to students for 20 dollars each. Three hundred students are willing to buy them at that price. For every 5 dollar increase in price, there are 30 fewer students willing to buy the calculator. What selling price will produce the maximum revenue and what will the maximum revenue be?

$R = (20+5x)(300-30x)$

Complete the square

$= 6000 - 600x + 1500x - 150x^2$

$= -150x^2 + 900x + 600$

$= -150(x^2 - 6x + 9 - 9) + 600$

$= -150(x-3)^2 + 1950$

Homework: p. 271 #16, 23, 24 & p. 313 #17c