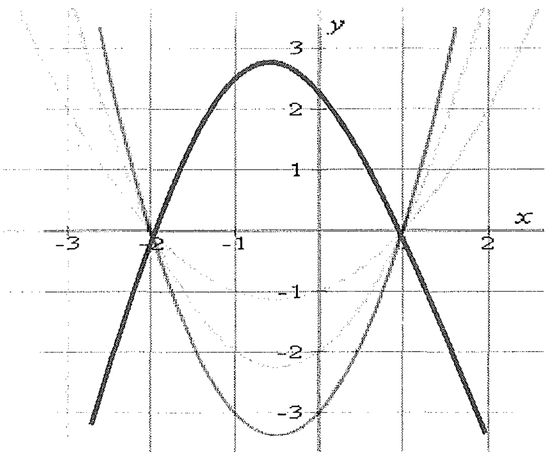
Day10-MCR3U

Families of Quadratic Functions

The graphs of 4 quadratic functions are shown below. Because they share key characteristics, we say that they belong to the same family.



How does each graph compare to the others? List as many similarities as possible.

- same zeros
- same axis of symmetry.
- Same shape (parabola)

Do you think this pair of functions should belong to the same family? Explain.

$y = 3(x+5)(x-2)$ and $y = -2(x-5)(x+2)$

zeros: $x = -5, 2$ zeros: $x = 5, -2$

No. They do not. They have different zeros and hence vertex.

Write two quadratic functions that belong to the same family as $y = (x-1)(x-6)$.

$y = 5(x-1)(x-6)$
$y = -2(x-1)(x-6)$
answers may vary.

$y = a(x-2)^2 + 4$ is a general equation for a family of quadratic functions with vertex (2,4). Each member has vertex (2, 4) but we can vary the value of "a" to create different quadratic functions

Write two quadratic functions that belong to the same family as $y = 3(x-1)^2 + 2$.

$y = -(x-1)^2 + 2$
$y = 2(x-1)^2 + 2$
answers may vary. vertex must be (1, 2)

$y = ax^2 + bx + 4$ is a general equation for a family of quadratic functions with y-intercept of 4. Each member has the same y-intercept but we can vary "a"

Write two quadratic functions that belong to the same family as $y = 3x^2 + 4$.

$y = 5x^2 + 4$ $\qquad$ $y = -x^2 + 4$

Example 1: The zeros of a quadratic function are 3 and -1.

a. Determine an equation in factored form for this family. $y = a(x-3)(x+1),\ a \neq 0$

b. Determine the equation of the family member whose graph has a y-intercept of -15. $(0, -15)$

$-15 = a(0-3)(0+1)$

$-15 = -3a$

$a = 5$

$\therefore y = 5(x-3)(x+1)$

Example 2: Find the equation in standard form of the quadratic function with zeros 2 and -5 and point (3, -16) is on the parabola.

$y = a(x-2)(x+5)$

$-16 = a(3-2)(3+5)$

$-16 = 8a$

$a = \frac{-16}{8}$

$= -2$

$\therefore y = -2(x-2)(x+5)$

this is in factored form

Expand and Simplify.

$\therefore y = -2(x^2+5x-2x-10) = -2(x^2+3x-10)$

$= -2x^2 - 6x + 20$

Example 3: Find the equation in standard form of the quadratic function that passes through (2, 5) if the roots are $1 + \sqrt{5}$ and $1 - \sqrt{5}$

$y = a(x-1-\sqrt{5})(x-1+\sqrt{5})$

$y = a(x^2-2x-4)$ sub $x=2$, $y=5$

$5 = a(2^2-2(2)-4)$

$5 = -4a$

$a = \frac{5}{-4}$

$\therefore y = \frac{-5}{4}(x^2-2x-4)$

$\therefore y = \frac{-5}{4}x^2 + \frac{5}{2}x + 5$

NOTE: $(x-1-\sqrt{5})(x-1+\sqrt{5})$

DOS

$= (x-1)^2 - (\sqrt{5})^2$

$= (x-1)(x-1) - 5$

$= x^2 - 2x + 1 - 5$

$= x^2 - 2x - 4$

Practice: Finding Equations of Quadratic Functions Given Two Roots and One Point

1. Find the equation in standard form of the quadratic function that passes through

a) (0, -9) if the zeros are $3 + \sqrt{10}$ and $3 - \sqrt{10}$

$y = a(x - 3 - \sqrt{10})(x - 3 + \sqrt{10})$

$y = a((x-3)^2 - 10)$

$y = a(x^2 - 6x + 9 - 10)$

$y = a(x^2 - 6x - 1)$

$x = 0 \quad y = -9$

$-9 = a(-1)$

$-9 = -a$

$a = 9 \quad \therefore y = 9(x^2 - 6x - 1)$

$= 9x^2 - 54x - 9$

b) (3, -16) if the zeros are $2 + \sqrt{5}$ and $2 - \sqrt{5}$

$y = a(x - 2 - \sqrt{5})(x - 2 + \sqrt{5})$

$y = a((x-2)^2 - 5)$

$y = a(x^2 - 4x + 4 - 5)$

$y = a(x^2 - 4x - 1)$ sub $x = 3$, $y = -16$

$-16 = a(3^2 - 4(3) - 1)$

$-16 = a(-4)$

$a = 4 \quad \therefore y = 4(x^2 - 4x - 1)$

$= 4x^2 - 16x - 4$

c) (-4, 9) if the zeros are $-2 + \sqrt{7}$ and $-2 - \sqrt{7}$

$y = a((x+2)^2 - 7)$

$y = a(x^2 + 4x + 4 - 7) = a(x^2 + 4x - 3)$

$9 = a((-4)^2 + 4(-4) - 3)$

$9 = -3a$

$a = -3$

$y = -3(x^2 + 4x - 3)$

$= -3x^2 - 12x + 9$

d) (4, 6) if the zeros are $-6 + \sqrt{90}$ and $-6 - \sqrt{90}$

$y = a(x + 6 - \sqrt{90})(x + 6 + \sqrt{90})$

$= a((x+6)^2 - 90)$

$y = a(x^2 + 12x - 54)$

sub $y = 6 \quad x = 4$

$6 = a(16 + 48 - 54)$

$6 = 10a \Rightarrow a = \frac{3}{5}$

$y = \frac{3}{5}(x^2 + 12x - 54)$

$= \frac{3}{5}x^2 + \frac{36}{5}x - \frac{162}{5}$

Homework: p. 192 #4, 8, 9

1a) f(x) = $9x^2 - 54x - 9$

b) f(x) = $4x^2 - 16x - 4$

c) f(x)= $-3x^2 - 12x + 9$

d) f(x) = $\frac{3}{5}x^2 + \frac{36}{5}x - \frac{162}{5}$