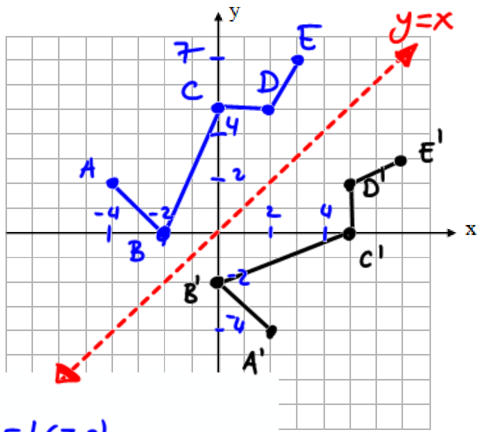
INVERSE OF A RELATION

INVESTIGATE:

a) Plot the points: A(–4, 2) B(–2, 0), C(0, 5), D(2, 5), E(3, 7)

b) Join the points in order, from A to E, using straight line segments.



c) State the Domain (D) and Range (R) of this function.

D = $\{x \in \mathbb{R} \mid -4 \leq x \leq 3\}$

R = $\{y \in \mathbb{R} \mid 0 \leq y \leq 7\}$

d) Interchange the Domain and Range and re-plot the points.

A(–4, 2) becomes A' (2, –4

B(-2,0) → B'(0,-2) E(3,7) → E'(7,3)
C(0,5) → C'(5,0)
D(2,5) → D'(5,2)

e) Again, join the points in order from A' to E'.

f) State the Domain and Range of the new relation.
*This is called the **INVERSE** relation of the original.*

D = $\{x \in \mathbb{R} \mid 0 \leq x \leq 7\}$

R = $\{y \in \mathbb{R} \mid -4 \leq y \leq 3\}$

g) Is the new relation also a function? Why or why not?
No, because it does not pass VLT

h) Graph the line **y = x**

i) How does the original function and its inverse seem to be related to the line y = x?

It appears that the original function has been reflected about the $y=x$ function.

INVERSE OF A LINEAR FUNCTION

INVESTIGATE:

a) Graph the linear function: **y = 2x + 4**

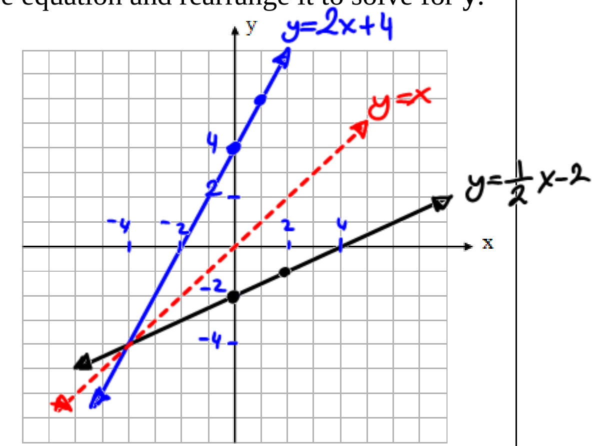
b) Interchange the x and y in the above equation and rearrange it to solve for **y**.
Step 1: x = 2y + 4

$\frac{x-4}{2} = \frac{2y}{2}$

$\frac{1}{2}x - 2 = y$

Rearrange

$y = \frac{1}{2}x - 2$



c) The result is the inverse equation for the original function: **y = 2x + 4:**

$y = \frac{1}{2}x - 2$

d) Is the inverse also a function? If so, what type?

Yes, it's also linear.

e) Graph the inverse and state how it is related to the original function and the line y = x.

reflected about $y=x$

g) State the Domain and Range for both y = 2x + 4 **and** for its inverse.

Original
D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R}\}$

Inverse
D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R}\}$

h) Find the inverse equation for the linear function: $y = -\frac{2}{3}x + 6$

$x = -\frac{2}{3}y + 6$

$3(x-6) = -\frac{2}{3}y \cdot 3$

$\frac{3x-18}{-2} = \frac{-2y}{-2}$

$\boxed{y = \frac{-3}{2}x + 9}$

INVESTIGATE: Inverse of a Quadratic Function

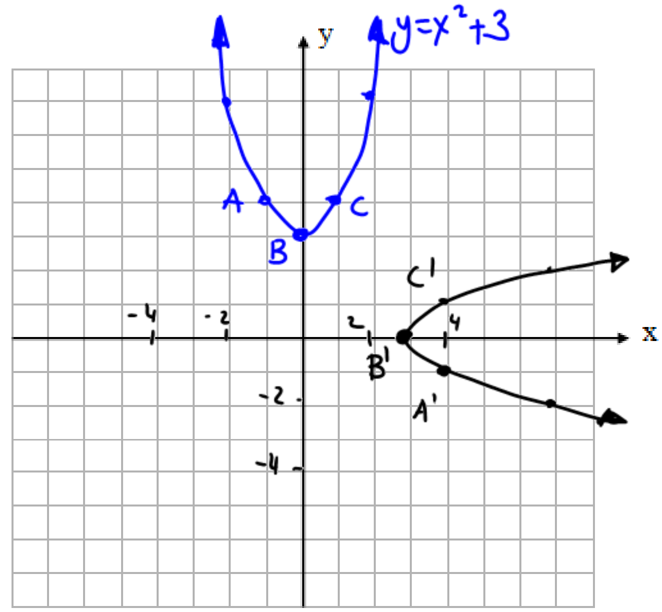
a) Graph the quadratic function: $\mathbf{y = x^2 + 3}$

b) State its Domain and Range.

$A(-1, 4) \to A'(4, -1)$
$B(0, 3) \to B'(3, 0)$
$C(1, 4) \to C'(4, 1)$

D = $\{x \in \mathbb{R}\}$

R = $\{y \in \mathbb{R} \mid y \geq 3\}$



c) Graph the inverse of this function by interchange x and y values for each point.

d) Is the inverse a function? No, doesn't pass VLT

e) State the Domain and Range for the Inverse.

D = $\{x \in \mathbb{R} \mid x \geq 3\}$

R = $\{y \in \mathbb{R}\}$

f) Find the Inverse Equation by interchanging x and y in the original equation and isolating **y**.

$\mathbf{x = y^2 + 3}$

$\sqrt{x-3} = \sqrt{y^2}$

$\mp\sqrt{x-3} = y$

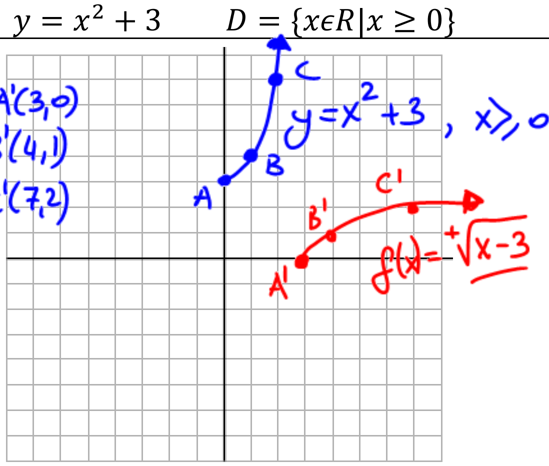
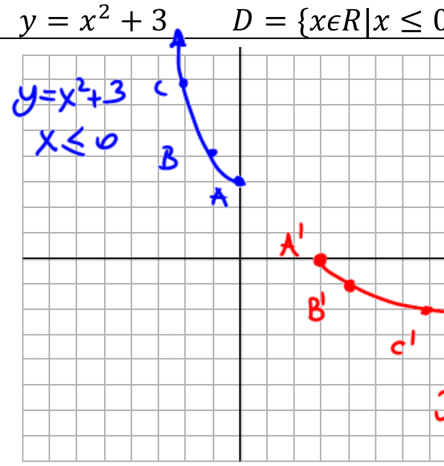
$y = +\sqrt{x-3}$ or $y = -\sqrt{x-3}$

$f^{-1}(x) = \sqrt{x-3}$ or $f^{-1}(x) = -\sqrt{x-3}$

If a relation is a function, the notation: f(x) may be used. If a function's inverse is also a function, the notation: $f^{-1}(x)$ is used. Note that f^{-1} is not an exponent; therefore, it is not 1/f

g) Sometimes, the inverse of a function is not also a **function**. In these cases, we **restrict** the domain of the **original** function so that its reflection in the line y = x is also a function.

For $\mathbf{y = x^2 + 3}$ the domain would be: $\{x\epsilon R | x \geq 0\}$. We are restricting the x values that are less than 0 so that the inverse function can pass the VLT test. In other words, when you graph the function, just draw the right arm of the parabola because it is where the x values are greater than or equal to 0.

a) Restrict the left arm, then inverse the function $y = x^2 + 3 \quad D = \{x\epsilon R \mid x \geq 0\}$	a) Restrict the right arm, then inverse the function $y = x^2 + 3 \quad D = \{x\epsilon R \mid x \leq 0\}$
$A(0,3) \to A'(3,0)$ $B(1,4) \to B'(4,1)$ $C(2,7) \to C'(7,2)$ 	 $A(0,3) \to A'(3,0)$ $B(-1,4) \to B'(4,-1)$ $C(-2,7) \to C'(7,-2)$

Practice

1. Find the inverse for each relation.

a) {(1, -3), (-2, 3), (5, 1), (6, 4)}

inverse {(-3,1), (3,-2), (1,5), (4,6)}

b) {(-5, 7), (-6, -8), (1, -2), (10, 3)}

inverse {(7,-5), (-8,-6), (-2,1), (3,10)}

2) Find an equation for the inverse for each of the following relations.

a) y = 3x + 2

$x = 3y + 2$

$\frac{x-2}{3} = \frac{3y}{3}$

$y = \frac{1}{3}x - \frac{2}{3} \Rightarrow f^{-1}(x) = \frac{1}{3}x - \frac{2}{3}$

b) y = -5x - 7

$x = -5y - 7$

$\frac{x+7}{-5} = \frac{-5y}{-5}$

$y = \frac{-1}{5}x - \frac{7}{5}$

$f^{-1}(x) = \frac{-1}{5}x - \frac{7}{5}$

c) $y = \frac{3}{4}x + 5$

$x = \frac{3}{4}y + 5$

$4 \cdot (x-5) = \frac{3}{4}y \cdot 4$

$\frac{4x}{3} - \frac{20}{3} = \frac{3y}{3}$

$y = \frac{4}{3}x - \frac{20}{3}$

$f^{-1}(x) = \frac{4}{3}x - \frac{20}{3}$

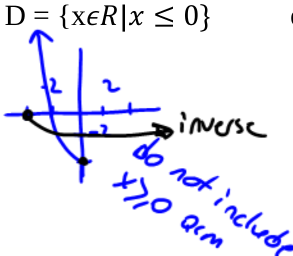
d) $y = x^2 - 4$ D = {x$\epsilon R | x \leq 0$}

$x = y^2 - 4$

$\sqrt{x+4} = \sqrt{y^2}$

$\mp\sqrt{x+4} = y$

We only pick $y = -\sqrt{x+4}$ b/c of Domain of Original Function

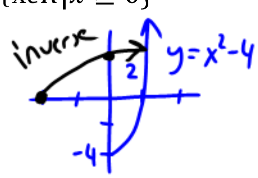


d) $y = x^2 - 4$ D = {x$\epsilon R | x \leq 0$}

e) $y = x^2 - 4$ D = {x$\epsilon R | x \geq 0$}

$y = +\sqrt{x+4}$

$f^{-1}(x) = \sqrt{x+4}$



e) $y = x^2 - 4$ D = {x$\epsilon R | x \geq 0$}

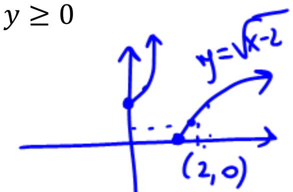
f) $y = \sqrt{x-2}, y \geq 0$

$(x)^2 = (\sqrt{y-2})^2$

$x^2 = y - 2$

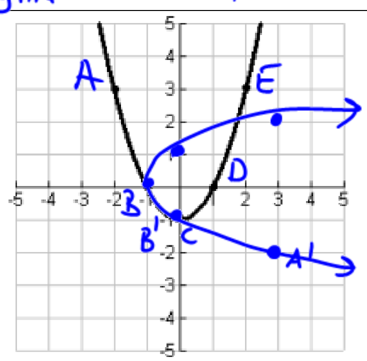
$\boxed{x^2 + 2 = y}$

$f^{-1}(x) = x^2 + 2 \quad x \geq 0$



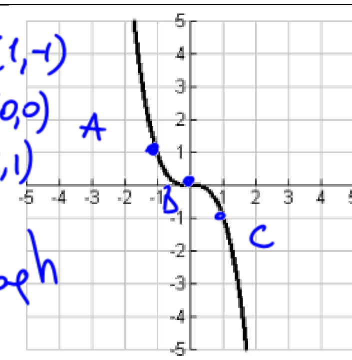
f) $y = \sqrt{x-2}, y \geq 0$

A'(3,-2)
B'(0,-1)
C'(-1,0)
D(0,1)
E(3,2)

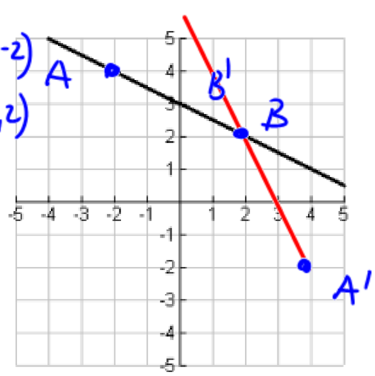


A(-1,1) → A'(1,-1)
B(0,0) → B'(0,0)
C(1,-1) → C'(-1,1)

same graph



A(-2,4) → A'(4,-2)
B(2,2) → B'(2,2)



A'(0,0)
B'(1,1)
C'(2,4)