6.2: Parallel and Perpendicular Lines

Two lines can be said to be parallel or perpendicular.

Two lines are parallel if they have the same slope.

Two lines are perpendicular if the slopes are negative reciprocals of each other. ($m_1 \times m_2 = -1$) → non horizontal non vertical.

Investigation

1) Go to the following site: https://www.geogebra.org/m/cSYbC98W

2) Which pair of lines are parallel? BLUE & GREEN

3) Which line is perpendicular to the others? RED.

4) Complete the following table. Record the slopes of all three lines. Drag points A, B, C and/or D to change their slopes. Record the new slopes values. Repeat.

Example	Slope of the GreenLine	Slope of the Blue Line	Slope of the Red Line
1	$-2/3$	$-2/3$	$3/2$
2	$-147/92$	$-147/92$	$92/147$
3	$104/219$	$104/219$	$-219/104$
4	1 or $1/1$	1 or $1/1$	-1 or $-1/1$
5	0	0	undefined.

5) What do you notice about the slopes of the parallel lines? They are the same

6) What do you notice about the slopes of perpendicular lines? $m_1 \times m_2 = -1$ for eg 1 to 4

They are negative reciprocals of each other.

By comparing the slopes of the pairs of lines that were parallel, develop a rule for the slopes of parallel lines.

If lines are parallel, then slopes are same. but y-intercepts are different.

By comparing the slopes of the pairs of lines that were perpendicular, develop a *rule* for the slopes of Perpendicular Lines..

If lines are perpendicular, then $m_1 \times m_2 = -1$

SUMMARY

PARALLEL Lines:	PERPENDICULAR Lines:
• Have the same slope • But differenty-intercepts • Are always a constant distance apart	• Meet at a right angle • The slopes are NEGATIVE RECIPROCAL of each other. • If $m_1 = \frac{a}{b}$ then $m_2 = -\frac{b}{a}$

Ex. 1) Which of the following pairs of lines are parallel?

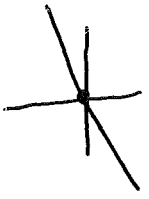
a) $y = 2x + 3$ and $y = 5x + 3$ $m_1 = 2 \quad m_2 = 5$ They are different ⇒ NOT PARALLEL	b) $y = -7x - 9$ and $y = -7x - 11$ $m_1 = -7 \quad m_2 = -7$ ∴ They are same ⇒ PARALLEL
c) $y = 6x + 1$ and $y = -6x + 4$ $m_1 = 6 \quad m_2 = -6$ They are different ⇒ NOT PARALLEL	d) $y = -3x - 2$ and $y = -3x$ $m_1 = -3 \quad m_2 = -3$ ∴ They are same ⇒ PARALLEL

Ex. 2) Which of the following pairs of lines are perpendicular?

a) $y = 2x + 7$ and $y = \frac{1}{2}x + 9$ $m_1 = 2 \quad m_2 = \frac{1}{2} \quad m_1 \times m_2 \neq -1$ ∴ NOT PERPENDICULAR.	b) $y = 5x + 1$ and $y = -5x - 3$ $m_1 = 5 \quad m_2 = -5$ $m_1 \times m_2 \neq -1$ ⇒ NOT PERPENDICULAR
c) $y = 9x + 8$ and $y = -\frac{1}{9}x + 5$ $m_1 = 9 \quad m_2 = -\frac{1}{9} \quad m_1 \times m_2 = -1$ ∴ Line 1 ⊥ line 2	d) $y = -\frac{2}{3}x - 11$ and $y = \frac{3}{2}x - 16$ $m_1 = -\frac{2}{3} \quad m_2 = \frac{3}{2}$ $m_1 \times m_2 = -1$ ⇒ ⊥

Ex. 3) Find an equation of a line that is parallel to $y = -5x + 1$ and passes through the point $(0, 0)$.

$m = -5$ $\quad b = 0$



$y = -5x$

Ex. 4) Find an equation of a line that is parallel to $y = 4x - 3$ and has the same y-intercept as $y = 3x - 9$.

$m = 4$ $\quad b = -9$

$y = 4x - 9$

Ex . 5) Find an equation of a line that is perpendicular *to* $y = \frac{1}{3}x - 2$ and passes through $(0, 3)$.

$m = \frac{1}{3}$

$m_{\perp} = -3$, $b = 3$ $\quad y = -3x + 3$

Ex. 6) Find an equation of a line that is perpendicular to $y = 4x + 2$ and has a y-intercept of 5

$m = 4$

$m_{\perp} = -\frac{1}{4}$, $b = 5$ $\quad y = -\frac{1}{4}x + 5$

Ex. 7) Find an equation of a line that is perpendicular to $y = 2x + 1$ and has the same y-intercept as $y = 6x - 2$.

$m = 2$

$m_{\perp} = -\frac{1}{2}$, $b = -2$ $\quad y = -\frac{1}{2}x - 2$

Seatwork: worksheets

Mark

Parallel & Perpendicular Lines

Date: 5 Dec 2018

Using the points given below, determine the slope of the line passing through the points, and determine which pairs of lines are parallel and which pairs are perpendicular.

Notation: If AB is parallel to CD, we write AB || CD.

If AB is perpendicular to CD, we write AB ⊥ CD.

Recall: Slope: $m_{AB} = \frac{y_B - y_A}{x_B - x_A}$ OR $m = \frac{y_2 - y_1}{x_2 - x_1}$

(x_1, y_1)	(x_2, y_2)	Slope (Steps)	Slope
A (–4,7)	B (5,8)	$m_{AB} = \frac{8-7}{5-(-4)} = \frac{1}{5+4}$	$\frac{1}{9}$
C (–4,4)	D (–1,5)	$m_{CD} = \frac{5-4}{-1+4} = \frac{1}{3}$	$\frac{1}{3}$
E (1,10)	F (2,7)	$m_{FE} = \frac{7-10}{2-1} = \frac{-3}{1}$	$\frac{-3}{1}$
G (7,–4)	H (10,2)	$m_{GH} = \frac{2+4}{10-7} = \frac{6}{3}$	2
I (6,12)	J (9,9)	$m_{IJ} = \frac{9-12}{9-6} = \frac{-3}{3} = -1$	–1
K (2,1)	L (6,2)	$m_{KL} = \frac{2-1}{6-2} = \frac{1}{4}$	$\frac{1}{4}$
M (–3,–3)	N (–2,–1)	$m_{MN} = \frac{-1+3}{-2+3} = \frac{2}{1}$	2
O (–1,–4)	P (4,–6)	$m_{OP} = \frac{-6+4}{4+1} = \frac{-2}{5}$	$\frac{-2}{5}$
Q (–8,6)	R (–4,10)	$m_{QR} = \frac{10-6}{-4+8} = \frac{4}{4}$	1
S (–5,2)	T (0,0)	$m_{ST} = \frac{0-2}{0+5} = \frac{-2}{5}$	$\frac{-2}{5}$

From the table above, list any lines that are parallel or perpendicular. Use proper notation.

Parallel lines: $\overline{OP} \parallel \overline{ST}$, $\overline{GH} \parallel \overline{MN}$

Perpendicular lines: $\overline{CD} \perp \overline{EF}$, $\overline{IJ} \perp \overline{QR}$

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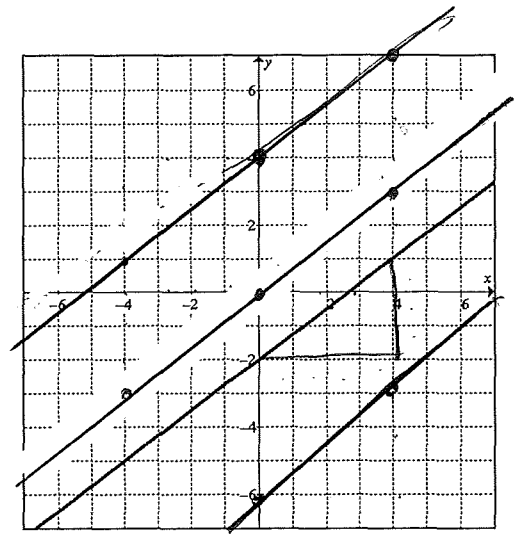
Parallel & Perpendicular Lines

Date:

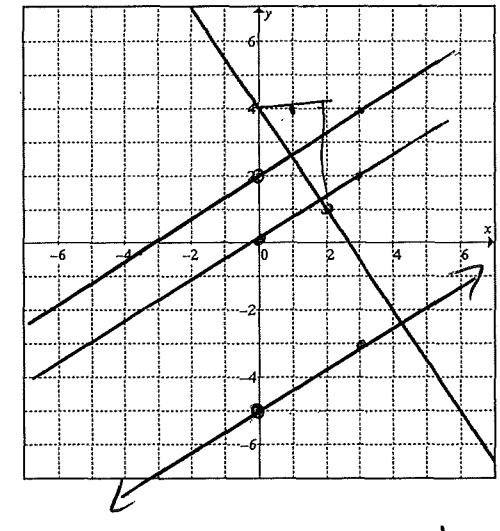
1. State the equation of the line shown on the Cartesian Plane given: $y = \frac{3}{4}x - 2$



   a) Draw 3 lines that are parallel to the given line having y-intercepts of -6, 0 and 4.

   b) Label each of the lines you have drawn with their respective equations.

2. State the equation of the line shown on the Cartesian Plane given: $y = -\frac{3}{2}x + 4$



   a) Draw 3 lines that are perpendicular to the given line having y-intercepts of -5, 0 and 2. $m_{\perp} = \frac{2}{3}$

   b) Label each of the lines you have drawn with their respective equations.

3. Beside each of the lines below, give its slope. Hint: "x-int" in the questions below is short for "x-intercept". *Work for these questions may be done on scrap paper.*

   a) The line $y = -2x - 1$ -2

   b) The line through (2,4) and (4,5) $\frac{1}{2}$

   c) The line with x-int 5 and y-int 3 $-\frac{3}{5}$ (5,0) (0,3)

   d) The line parallel to $y = 7 - \frac{3}{5}x$ $-\frac{3}{5}$

   e) The line with rise of 5 and run of 2 $\frac{5}{2}$

   f) The line $y = x + 1$ 1

   g) The line through (–3,1) and (1,5) $\frac{4}{4} = 1$

   h) The line $y = \frac{2}{3}x + 5$ $\frac{2}{3}$

   i) The line with rise of –2 and run 3 $\frac{-2}{3}$

   j) The line $\perp$ to $y = -\frac{3}{4}x - 1$ $\frac{4}{3}$

   k) The line through (4,–4) and (2,–7) $\frac{3}{2}$

   l) The line with x-int –2 and y-int –1 $-\frac{1}{2}$

In the space provided, list all pairs of lines from #3 above which are either parallel or perpendicular.

Parallel lines: c ∥ d f ∥ g

Perpendicular lines: a ⊥ b i ⊥ k

Answers:

-2 ; $\frac{1}{2}$; $-\frac{3}{5}$; $-\frac{3}{5}$; $\frac{5}{2}$; 1 ; 1 ; $\frac{2}{3}$; $-\frac{2}{3}$; $\frac{4}{3}$; $\frac{3}{2}$; $-\frac{1}{2}$

c ∥ d ; f ∥ g ; a ⊥ b ; i ⊥ k

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