

DECOMPOSITION METHOD

Factor $2x^2 + x - 6$

3 -4

The two numbers that add to +1 and multiply to -12 are ...

$$= 2x^2 + 3x - 4x - 6$$

$$= x(2x + 3) - 2(2x + 3)$$

$$= (2x + 3)(x - 2)$$

Split the middle term using the two numbers you found, mx and nx.

Factor these 4 terms by grouping. (i.e. common factor the first two terms, common factor the last two terms - Remember: the two binomials in the brackets should always be the same).

Common factor the binomials in the brackets.

AUSTRALIAN METHOD

Example 1: Factor $2x^2 + x - 6$

3 -4

The two numbers that add to +1 and multiply to -12 are ...

$$= \frac{(2x + 3)(2x - 4)}{2}$$

$$= \frac{(2x + 3)\cancel{2}(x - 2)}{\cancel{2}} \rightarrow (2x + 3)(x - 2)$$

Write the following terms in this form:

$$\frac{(ax + m)(ax + n)}{a}$$

Common factor the two sets of brackets.

Reduce the fraction.

Example: Factor $2x^2 + 11x + 5$

m and n must ... + to 11, x to 10

m and n are +1, +10

$$2x^2 + 11x + 5$$

$$= 2x^2 + x + 10x + 5$$

$$= x(2x + 1) + 5(2x + 1)$$

$$= (2x + 1)(x + 5)$$

Decomp.

Example: Factor $6x^2 - 7x + 2$

m and n must ... + to -7, x to 12

m and n are -3, -4

AUS $6x^2 - 7x + 2$

$$= \frac{(6x - 3)(6x - 4)}{6}$$

$$= \frac{\cancel{2}(2x - 1)\cancel{2}(3x - 2)}{\cancel{6}}$$

$$= (2x - 1)(3x - 2)$$

DEC

$$6x^2 - 3x - 4x + 2$$

$$= 3x(2x - 1) - 2(2x - 1)$$

$$= (2x - 1)(3x - 2)$$

Example 4: Factor $4x^2 - 4x - 3$

m and n must ... + to -4, x to -12

m and n are +2, -6

DEC $4x^2 - 4x - 3$ OR

$$4x^2 - 6x + 2x - 3$$

$$= 2x(2x - 3) + (2x - 3)$$

$$= (2x - 3)(2x + 1)$$

AUS

$$= \frac{(2x - 6)(4x + 2)}{4}$$

$$= \frac{\cancel{2}(2x - 3)\cancel{2}(2x + 1)}{\cancel{4}}$$

$$= (2x - 3)(2x + 1)$$

Example 5: Factor $8x^2 + 2x - 3$

m and n must ... + to +2, x to -24

m and n are -4, +6

AUS $8x^2 + 2x - 3$

$$= \frac{(8x - 4)(8x + 6)}{8}$$

$$= \frac{4(2x - 1)\cancel{2}(4x + 3)}{\cancel{8}}$$

$$= (2x - 1)(4x + 3)$$

Example 6: Factor $4x^2 - 27x + 18$

m and n must ... + to -27, x to 72

m and n are -3, -24

$$4x^2 - 27x + 18$$

$$\begin{aligned} &= 4x^2 - 3x - 24x + 18 & | &= \frac{(4x-3)(4x-24)}{4} \\ &= x(4x-3) - 6(4x-3) & | &= \frac{(4x-3)\cancel{4}(x-6)}{4} \\ &= (4x-3)(x-6) & | &= \frac{\cancel{4}}{4} \\ & & | &= (4x-3)(x-6) \end{aligned}$$

Example 7: Factor $9x^2 - 6x - 8$

m and n must ... + to -6, x to -72

m and n are +6, -12

$$9x^2 - 6x - 8$$

$$\begin{aligned} &= 9x^2 - 12x + 6x - 8 & | &= \frac{(9x-12)(9x+6)}{9} \\ &= 3x(3x-4) + 2(3x-4) & | &= \frac{\cancel{3}(3x-4)\cancel{3}(3x+2)}{\cancel{9}} \\ &= (3x-4)(3x+2) & | &= \frac{\cancel{9}}{\cancel{9}} \\ & & | &= (3x-4)(3x+2) \end{aligned}$$

Example 8: Factor $6x^2 - 7x - 3$

m and n must ... + to -7, x to -18

m and n are +2, -9

$$6x^2 - 7x - 3$$

$$\begin{aligned} &6x^2 - 9x + 2x - 3 & | &= \frac{(6x-9)(6x+2)}{6} \\ &= 3x(2x-3) + (2x-3) & | &= \frac{\cancel{3}(2x-3)\cancel{2}(3x+1)}{\cancel{6}} \\ &= (2x-3)(3x+1) & | &= \frac{\cancel{6}}{\cancel{6}} \\ & & | &= (2x-3)(3x+1) \end{aligned}$$

Example 9: Factor $4x^2 - 13x - 12$

m and n must ... + to -13, x to -48

m and n are 3, -16

$$4x^2 - 13x - 12$$

$$\begin{aligned} &4x^2 - 16x + 3x - 12 & | &= \frac{(4x-16)(4x+3)}{4} \\ &= 4x(x-4) + 3(x-4) & | &= \frac{4(x-4)(4x+3)}{4} \\ &= (x-4)(4x+3) & | &= (x-4)(4x+3) \end{aligned}$$

Example 10: Factor $5x^2 + 13x - 6$

m and n must ... + to 13, x to -30

m and n are -2, 15

$$5x^2 + 13x - 6$$

$$\begin{aligned} &5x^2 - 2x + 15x - 6 & | &= \frac{(5x-2)(5x+15)}{5} \\ &= x(5x-2) + 3(5x-2) & | &= \frac{(5x-2)\cancel{5}(x+3)}{\cancel{5}} \\ &= (5x-2)(x+3) & | &= \frac{\cancel{5}}{\cancel{5}} \\ & & | &= (5x-2)(x+3) \end{aligned}$$

Example 11: Factor $8x^2 - 14x + 3$

m and n must ... + to -14, x to 24

m and n are -2, -12

$$8x^2 - 14x + 3$$

$$\begin{aligned} &8x^2 - 12x - 2x + 3 & | &= \frac{(8x-12)(8x-2)}{8} \\ &= 4x(2x-3) - 1(2x-3) & | &= \frac{\cancel{4}(2x-3)\cancel{2}(4x-1)}{\cancel{8}} \\ &= (2x-3)(4x-1) & | &= \frac{\cancel{8}}{\cancel{8}} \\ & & | &= (2x-3)(4x-1) \end{aligned}$$

Tricky Practice

3, -5
1) $3p^2 - 2p - 5 = 3p^2 + 3p - 5p - 5$
 $= 3p(p+1) - 5(p+1)$
 $= (p+1)(3p-5)$

12
3) $3n^2 - 8n + 4$
 $= 3n^2 - 2n - 6n + 4$
 $= n(3n-2) - 2(3n-2)$
 $= (3n-2)(n-2)$

5) $2v^2 + 11v + 5$

M	A	N
10	11	1, 10

 $= \frac{(2v+1)(2v+5)}{2}$
 $= \frac{(2v+1)(2)(v+5)}{2} \Rightarrow (2v+1)(v+5)$

7) $7a^2 + 53a + 28$

M	A	N
19	53	4, 49

 $= 7a^2 + 4a + 49a + 28$
 $= a(7a+4) + 7(7a+4)$
 $= (7a+4)(a+7)$

9) $15n^2 - 27n - 6$

M	A	N
-90	-27	3, -30

 $= 15n^2 - 30n + 3n - 6$
 $= 15n(n-2) + 3(n-2)$
 $= (n-2)(15n+3)$
 $= (n-2)(3)(5n+1)$

11) $4n^2 - 15n - 25$

M	A	N
-100	-15	5, -20

 $= 4n^2 - 20n + 5n - 25$
 $= 4n(n-5) + 5(n-5)$
 $= (n-5)(4n+5)$

15) $6x^2 + 37x + 6$

M	A	N
36	37	1, 36

 $= 6x^2 + x + 36x + 6$
 $= x(6x+1) + 6(6x+1)$
 $= (6x+1)(x+6)$

17) $6n^2 + 5n - 6$

M	A	N
-36	5	-4, 9

 $= 6n^2 - 4n + 9n - 6$
 $= 2n(3n-2) + 3(3n-2)$
 $= (3n-2)(2n+3)$

2) $2n^2 + 3n - 9$
 $= 2n^2 - 3n + 6n - 9$
 $= n(2n-3) + 3(2n-3)$
 $= (2n-3)(n+3)$

4) $5n^2 + 19n + 12$

M	A	N
60	19	4, 15

 $= 5n^2 + 4n + 15n + 12$
 $= n(5n+4) + 3(5n+4)$
 $= (5n+4)(n+3)$

6) $2n^2 + 5n + 2$

M	A	N
4	5	1, 4

 $= \frac{(2n+1)(2n+4)}{2}$
 $= \frac{(2n+1)(2)(n+2)}{2} = (2n+1)(n+2)$

8) $9k^2 + 66k + 21$

M	A	N
189	66	3, 63

 $= 9k^2 + 3k + 63k + 21$
 $= 3k(3k+1) + 21(3k+1)$
 $= (3k+1)(3k+21)$
 $= (3k+1)(3)(k+7)$

10) $5x^2 - 18x + 9$

M	A	N
45	-18	3, -15

 $= 5x^2 - 15x - 3x + 9$
 $= 5x(x-3) - 3(x-3)$
 $= (x-3)(5x-3)$

12) $4x^2 - 35x + 49$

M	A	N
196	-35	7, 28

 $= 4x^2 - 28x - 7x + 49$
 $= 4x(x-7) - 7(x-7)$
 $= (x-7)(4x-7)$

16) $-6a^2 - 25a - 25$

M	A	N
150	-25	-10, 15

 $= -6a^2 - 15a - 10a - 25$
 $= -3a(2a+5) - 5(2a+5)$
 $= (2a+5)(-3a-5)$
 $= (2a+5)(-1)(3a+5)$

18) $16b^2 + 60b - 100$

M	A	N
-100	15	-5, 20

 $= 4(4b^2 + 15b - 25)$
 $= 4(4b^2 - 5b + 20b - 25)$
 $= 4[b(4b-5) + 5(4b-5)]$
 $= 4(4b-5)(b+5)$

Answers
(not in order):

$(7a+4)(a+7)$

$-(2a+5)(3a+5)$

$4(b+5)(4b-5)$

$3(3k+1)(k+7)$

$(n-4)(4n-1)$

$(n-5)(4n+5)$

$(2n+1)(n+2)$

$(2n+3)(3n-2)$

$(2n-3)(n+3)$

$(3n-2)(n-2)$

$(5n+4)(n+3)$

$3(5n+1)(n-2)$

$(3p-5)(p+1)$

$(2v+1)(v+5)$

$(x-7)(4x-7)$

$(x+6)(6x+1)$

$(3x-7)(2x+7)$

$(5x-3)(x-3)$