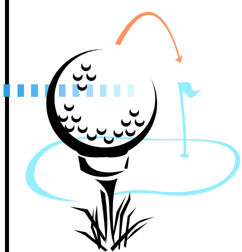
Solving Quadratic Equations by Using the Formula

Warm-Up:

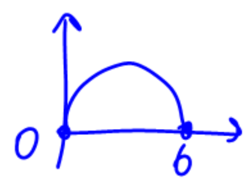


A. The height, h, in metres, of a golf ball t seconds after Billy Bob hits it with a club is described by $h = -5t^2 + 30t$. How long is the ball in the air?

$0 = -5t^2 + 30t$ GCF $= -5t$

$0 = -5t(t - 6)$

$-5t = 0$ $\boxed{t = 0}$ $t - 6 = 0$ $\boxed{t = 6}$



$\therefore$ 6 seconds

B. The height, h, in metres, of a football t seconds after Billy Bob throws it off the roof of a building is described by $h = -5t^2 + 20t + 25$. How long is the ball in the air?

$0 = -5t^2 + 20t + 25$

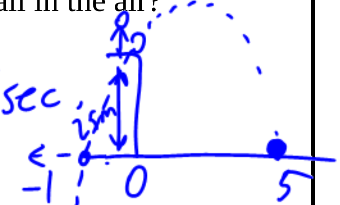
$0 = -5(t^2 - 4t - 5)$

$0 = -5(t + 1)(t - 5)$

$t + 1 = 0$ $\boxed{t = -1}$ $t - 5 = 0$ $\boxed{t = 5}$

M	A	N
-5	-4	1, -5

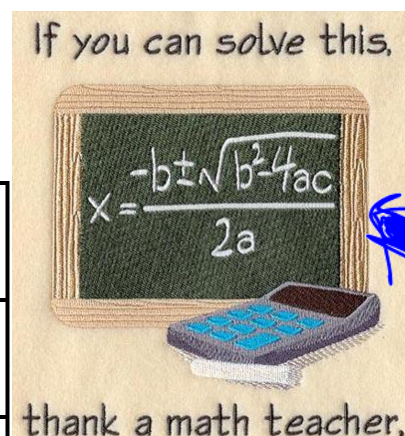
$\therefore$ 5 sec.



C. The height, h, in metres, of a baseball t seconds after Billy Bob hits it with a bat is described by $h = -5t^2 + 18t+1$. For how long is the ball in the air?

Standard form $y = -5t^2 + 20t + 25$ (a, b, c)

To find the zeros of $ax^2 + bx + c = 0$, you can use this Quadratic Formula:



Example B from above: **a** = -5 **b** = $+20$ **c** = $+25$

• substitute the values of a, b, and c into the formula (use brackets!)	$t = \frac{-(20) \pm \sqrt{(20)^2 - 4(-5)(25)}}{2(-5)}$
• use your calculator to determine the value under the square root sign	$t = \frac{(-20) \pm \sqrt{(900)}}{(-10)}$ 400+500
• take the square root (round to 1 decimal if required)	$t = \frac{(-20) \pm (30)}{(-10)}$
• split the equation into 2 equations – one with the '+' and one with the '–'	$t = \frac{(-20) + (30)}{(-10)}$ or $t = \frac{(-20) - (30)}{(-10)}$
• calculate the numerators	$t = \frac{(10)}{(-10)}$ or $t = \frac{(-50)}{(-10)}$
• divide to get your zeros	$t =$ -1 or $t =$ 5 ✓

Discriminant

Do you see b^2 - 4ac in the formula above? It is called the **Discriminant**, because it can "discriminate" between the possible types of answer:

- when b^2 - 4ac is positive, we get two Real roots (The graph has two x-intercepts)
- when it is zero we get just ONE real root (both answers are the same; the graph has one x-intercept)
- when it is negative we get no Real root

1. $-5x^2 + 18x + 1 = 0$

a = -5 b = 18 c = 1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-(18) \pm \sqrt{(18)^2 - 4(-5)(1)}}{2(-5)}$$

$$x_{1,2} = \frac{-18 \mp \sqrt{344}}{-10}$$

$$x_{1,2} = \frac{-18 \mp 18.5}{-10}$$

$$x_1 = \frac{-18 + 18.5}{-10} \qquad x_2 = \frac{-18 - 18.5}{-10}$$

$$x_1 = \frac{0.5}{-10} \qquad x_2 = \frac{-36.5}{-10}$$

$$\boxed{x_1 = -0.05} \qquad \boxed{x_2 = 3.65}$$

2. $-5x^2 + 30x = 0$

a = -5 b = 30 c = 0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-30 \mp \sqrt{30^2 - 4(-5)(0)}}{2(-5)}$$

$$x_{1,2} = \frac{-30 \mp \sqrt{900}}{-10}$$

$$x_{1,2} = \frac{-30 \mp 30}{-10}$$

$$x_1 = \frac{-30 + 30}{-10} \qquad x_2 = \frac{-30 - 30}{-10}$$

$$\boxed{x_1 = 0} \qquad \boxed{x_2 = 6}$$

3. $-3x^2 + x - 7 = 0$

a = -3 b = 1 c = -7

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-1 \mp \sqrt{1^2 - 4(-3)(-7)}}{2(-3)}$$

$$x_{1,2} = \frac{-1 \mp \sqrt{-83}}{-6}$$

→ can't square root a "-" number

∴ no solution.

- Notice when discriminant is - no solutions.

4. $x^2 - 6x + 9 = 0$

a = 1 b = -6 c = 9

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-(-6) \mp \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$$

$$x_{1,2} = \frac{6 \mp \sqrt{36 - 36}}{2}$$

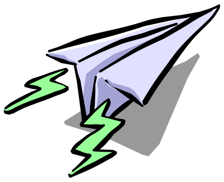
$$x_{1,2} = \frac{6 \mp 0}{2}$$

$$x_1 = \frac{6 + 0}{2} \qquad x_2 = \frac{6 - 0}{2}$$

$$x_1 = 3 \qquad x_2 = 3$$

∴ x = 3, when disc = 0, one solution.

5. A paper airplane follows a parabolic path with $h = -4t^2 + 11t + 3$, where h is height in metres, and t is time in seconds. Use the quadratic formula to determine how long it takes for the paper airplane to hit the ground. Verify your answer using factoring.



$0 = -4t^2 + 11t + 3$

$a = -4 \quad b = 11 \quad c = 3$

$x_{1,2} = \frac{-11 \mp \sqrt{(11)^2 - 4(-4)(3)}}{2(-4)} = \frac{-11 \mp \sqrt{169}}{-8} = \frac{-11 \mp 13}{-8}$

$x_1 = \frac{-11+13}{-8}$ $\boxed{x_1 = -0.25}$

$x_2 = \frac{-11-13}{-8}$ $\boxed{x_2 = 3}$

x_1

$\boxed{\therefore 3 \text{ sec}}$

6. The path of one freestyle aerial skier from the top of the kicker (i.e. the ramp) to the landing point can be modeled by $h = -0.2d^2 + 2.5d + 8$, where h is the height in metres above the landing point and d is the horizontal distance from the kicker.

a. When does the skier land?

$0 = -0.2d^2 + 2.5d + 8$

$a = -0.2 \quad b = 2.5 \quad c = 8$

$x_{1,2} = \frac{-2.5 \mp \sqrt{(2.5)^2 - 4(-0.2)(8)}}{2(-0.2)} = \frac{-2.5 \mp \sqrt{12.65}}{-0.4} = \frac{-2.5 \mp 3.6}{-0.4}$

$x_1 = \frac{-2.5+3.6}{-0.4}$ $\boxed{x_1 = -2.75}$

$x_2 = \frac{-2.5-3.6}{-0.4}$ $\boxed{x_2 = 15.25}$

$\therefore$ at 15.25 sec.

b. For how long is the skier above a height of 10 m?

$10 = -0.2d^2 + 2.5d + 8$

$0 = -0.2d^2 + 2.5d + 8 - 10$

$0 = -0.2d^2 + 2.5d - 2$

$a = -0.2 \quad b = 2.5 \quad c = -2$

$x_{1,2} = \frac{-2.5 \mp \sqrt{(2.5)^2 - 4(-0.2)(-2)}}{2(-0.2)}$

$x_{1,2} = \frac{-2.5 \mp 0.9}{-0.4}$

$x_1 = \frac{-2.5+0.9}{-0.4} = 4$

$x_2 = \frac{-2.5-0.9}{-0.4} = 8.5$

$\therefore 8.5 - 4 = 4.5$ sec.

c. Is it possible for the skier to reach a height of 20 m?

$25 = -0.2d^2 + 2.5d + 8$

$0 = -0.2d^2 + 2.5d - 17$

$a = -0.2 \quad b = 2.5 \quad c = -17$

$x_{1,2} = \frac{-2.5 \mp \sqrt{(2.5)^2 - 4(-0.2)(-17)}}{-0.4} = \frac{-2.5 \mp \sqrt{-7.35}}{-0.4}$ NO SOLUTION

$\therefore$ NOT POSSIBLE

it doesn't intercept with the graph.

6. The length of a rectangular garden is 6 more than the width. The area is $27m^2$. Use the quadratic formula to determine the dimensions of the garden. Verify your answer using factoring.

x+6	
A = 27	x

$27 = x(x+6)$

$27 = x^2 + 6x$

$0 = x^2 + 6x - 27$

$a = 1 \quad b = 6 \quad c = -27$

$x_{1,2} = \frac{-6 \mp \sqrt{6^2 - 4(1)(-27)}}{2(1)} = \frac{-6 \mp 12}{2}$

$x_1 = \frac{-6+12}{2} = 3$

$x_2 = \frac{-6-12}{2} = -9$

$\therefore \boxed{x = 3}$

CHECK

$0 = x^2 + 6x - 27$

$0 = (x-3)(x+9)$ ✓

$x - 3 = 0 \quad x = 3$

$x + 9 = 0 \quad x = -9$