We can now solve a quadratic equation by **graphing**, **factoring**, **completing the square** or the **quadratic formula**. We need to know how to apply these skills and use the most appropriate method in a given situation.

Example 1: Path of an Object

The formula $h = -\frac{1}{2}gt^2 + v_o t + h_o$ can be used to model the height of a projectile, where g is acceleration due to gravity, which is 9.8m/s² on Earth, v_o is the initial vertical velocity, in metres per second, and h_o is the initial height, in metres.

a. Create a model for the height of a toy rocket launched upward at 60m/s (v_o) from the top of a 3-m (h_o) platform.

$h = -\frac{1}{2}(9.8)t^2 + 60t + 3$

$h = -4.9t^2 + 60 + 3$

b. How long would the rocket take to fall to Earth, rounded to the nearest hundredth of a second?

$0 = -4.9t^2 + 60t + 3$

$a = -4.9 \quad b = 60 \quad c = 3$

$x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{-60 \mp \sqrt{(60)^2 - 4(-4.9)(3)}}{2(-4.9)} = \frac{-60 \mp \sqrt{3600 + 58.8}}{-9.8} = \frac{-60 \mp \sqrt{3658.8}}{-9.8}$

$x_{1,2} \doteq \frac{-60 \mp 60.5}{-9.8} \rightarrow x_1 = \frac{-60 + 60.5}{-9.8} \doteq 0.05$

$x_2 = \frac{-60 - 60.5}{-9.8} \doteq 12.3$

∴ It would take about 12.3 second for the rocket to fall to Earth.

c. What is the maximum height of the rocket, rounded to the nearest metre?

$h = -4.9t^2 + 60t + 3$

$= -4.9(t^2 - 12.24t) + 3 \qquad \frac{-12.24}{2} = -6.12 \qquad (-6.12)^2 = 37.45$

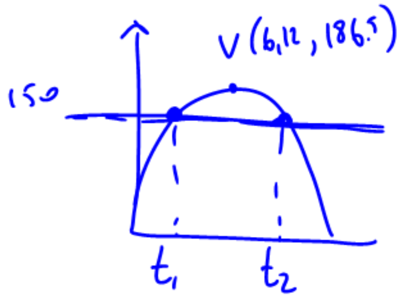
$= -4.9(t^2 - 12.24t + 37.45 - 37.45) + 3$

$= -4.9(t^2 - 12.24t + 37.45) + 183.5 + 3$

$= -4.9(t - 6.12) + 186.5$

∴ about 187m

d. Over what time interval is the height of the toy rocket greater than 150 m? Round to the nearest hundredth of a second.



$150 = -4.9t^2 + 60t + 3$

$0 = -4.9t^2 + 60t - 147$

$a = -4.9 \quad b = 60 \quad c = -147$

$t_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{-60 \mp \sqrt{(60)^2 - 4(-4.9)(-147)}}{2(-4.9)} = \frac{-60 \mp \sqrt{3600 - 2881.2}}{-9.8}$

$t_{1,2} = \frac{-60 \mp \sqrt{718.8}}{-9.8} = \frac{-60 \mp 26.8}{-9.8} \rightarrow t_1 = \frac{-60 + 26.8}{-9.8} = 3.39$

$t_2 = \frac{-60 - 26.8}{-9.8} = 8.86$

∴ The height of the toy is greater than 150m from about 3.39s to 8.86s.

Example 2: Consecutive Numbers
The product of two consecutive even numbers is 5624. What are the numbers?

Let "x" be one even number, therefore x+2 is the next one.

$x(x+2) = 5624$
$x^2 + 2x = 5624$
$x^2 + 2x - 5624 = 0$
$a=1 \quad b=2 \quad c=-5624$

$x_{1,2} = \frac{-b \mp \sqrt{b^2-4ac}}{2a} = \frac{-2 \mp \sqrt{2^2-4(1)(-5624)}}{2(1)} = \frac{-2 \mp \sqrt{22500}}{2}$

$= \frac{-2 \mp 150}{2}$

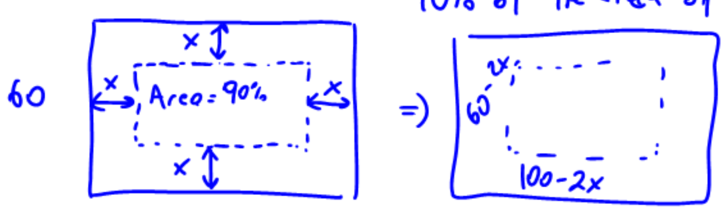
$x_1 = \frac{-2+150}{2} = 74$

$x_2 = \frac{-2-150}{2} = -76$

$\therefore$ The numbers are either 74, 76 or −76, −74

Example 3: Width of a Path
A rectangular park measures 100 m by 60 m. A path of constant width is to be paved around the perimeter. The mayor wants to be sure that the path does not reduce the area of the grass by more than 10%. What is the maximum allowable width of the path, rounded to the nearest tenth of a metre?

90% of the area of the park = 6000 x 90% = 5400



$5400 = (60-2x)(100-2x)$
$5400 = 6000 - 120x - 200x + 4x^2$
$0 = 4x^2 - 320x + 600$
$\frac{0}{4} = \frac{4(x^2-80x+150)}{4}$
$0 = x^2 - 80x + 150$

$a=1 \quad b=-8 \quad c=150$

$x_{1,2} = \frac{-b \mp \sqrt{b^2-4ac}}{2a} = \frac{80 \mp \sqrt{6400-4(1)(150)}}{2} = \frac{8 \mp \sqrt{5800}}{2}$

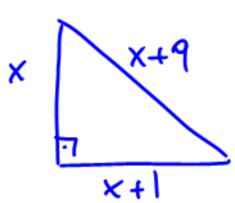
$= \frac{80 \mp 76.2}{2}$

$x_1 = \frac{80+76.2}{2} = 78.1$

$x_2 = \frac{80-76.2}{2} = 1.9$

$\therefore$

Example 4: Right Triangle
One leg of a right triangle is 1 cm longer than the other leg. The length of the hypotenuse is 9 cm greater than that of the shorter leg. Find the length of the three sides.



Pythagorean Theorem

$(x+9)^2 = x^2 + (x+1)^2$
$x^2 + 18x + 81 = x^2 + x^2 + 2x + 1$
$0 = 2x^2 + 2x + 1 - x^2 - 18x - 81$
$0 = x^2 - 16x - 80$
$0 = (x-20)(x+4)$
$x - 20 = 0 \quad x+4=0$
$x = 20 \quad x = -4$