Solving Linear Systems – I've Got Problems!

Practice Makes Progress!

1. Carl's Car Rental Company charges a flat rate of $20 plus 25¢ per km. Cally's Car
    Rental Company charges a flat rate of $10 plus 30¢ per km.

    a. Develop an equation to represent the cost of renting a car from each
        company. Let x represent the number of km driven and y represent the total
        cost.

$25¢ = \$0.25$

$30¢ = \$0.30$

  ① Carl's Car Rental Company $y = \underline{0.25x + 20}$
  ② Cally's Car Rental Company $y = 0.30x + 10$

solution

Where do the two lines intersect? What does this point represent?

① $y = 0.25x + 20$
② $y = 0.30x + 10$ sub ①→②

$0.30x + 10 \overset{-10}{=} 0.25x + 20^{-10}$
$0.30x \overset{-0.25x}{=} 0.25x + 10^{-0.25x}$
$\dfrac{0.05x}{0.05} = \dfrac{10}{0.05}$
$\boxed{x = 200}$

$y = 0.25x + 20$ sub $x = 200$
$= 0.25(200) + 20$
$= 50 + 20$ ∴ The POI is
$\boxed{y = 70}$ (200, 70)
                     Each company cost
                     the same for 70km
                     driven.

    b. **Sketch** the graphs and label the lines.

    c. Which company will be cheapest if you plan to drive
        300km?

You pay less with Carl's Car Rental Co
if you plan to drive 300km because
its graph grows slower compared to that of
Cally's after POI.
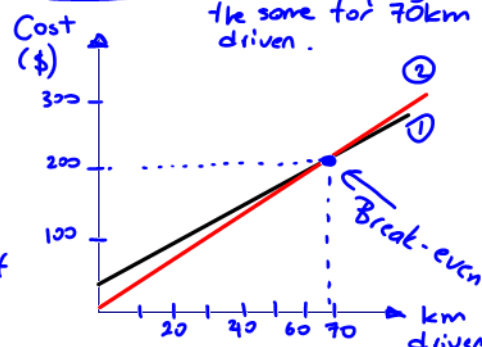
2. There are two competing video stores. Video Mania rents videos for $2 each plus a
    one-time membership fee of $10. Cool Movies rents videos for $2.50 each with NO
    membership fee. When will the cost be the same at either rental store?

    Let v be the number of videos rented
    Let c be the total charge

Video Mania → $c = 2v + 10$ ①
Cool Movie → $c = 2.5v$ ②

Sub ① in ②
$2v + 10^{-2v} = 2.5v^{-2v}$
Step 1
$\dfrac{10}{0.5} = \dfrac{0.5v}{0.5}$
$\boxed{v = 20}$

Step 2
$C = 2.5v \quad v = 20$
$= 2.5(20)$
$\boxed{C = 50}$

POI (50, 20)

Step 3
The cost will be
the same which is
$50 when 20 videos
rented.
(50, 20)

3.   When Chana rented a car for three days and drove 160km, the charge was $124.
     When she rented the same car for five days and drove 400km, the charge was $240.
     What was the charge per day and the charge per km?

     Let d be the cost per day   ① $3d + 160k = 124$
     Let k be the cost per km driven   ② $5d + 400k = 240$

Step 1 Rearrange ② in $y=mx+b$ form
to isolate d

$$5d + 400k \underset{-400k}{} = 240 \underset{-400k}{}$$

$$\frac{5d}{5} = \frac{240-400k}{5}$$

③ $\boxed{d = 48 - 80k}$

Step 2 sub ③ into ① where you see d

$3(48-80k)+160k=124$

$144 - 240k + 160k = 124$

$$144 \underset{-144}{} - 240k + 160k = 124 \underset{-144}{}$$

$$\frac{-80k}{-80} = \frac{-20}{-80}$$

$$\boxed{k = \tfrac{1}{4}}$$
$\downarrow$
$k = 0.25$
$= 25¢$

Step 3 $3d + 160k = 124$ $k = \tfrac{1}{4}$

$3d + 160(\tfrac{1}{4}) = 124$

$$3d + 40 \underset{-40}{} = 124 \underset{-40}{}$$

$$\frac{3d}{3} = \frac{84}{3}$$

$\boxed{d = 28}$

∴ The charge per km is 25¢
and per day is $28.

4.   Charlotte invested $800, part at 9% per year and the rest at 12% per year. After one
     year, the total interested earned was $79.50. How much did she invest at each rate?

     Let n be the amount invested at 9%   ① $n + t = 800$
     Let t be the amount invested at 12%   ② $0.09n + 0.12t = 79.50$

Step 1 Rearrange ① to isolate n

$n + t = 800$
③ $\boxed{n = 800 - t}$

Step 2 sub ③ into ②

$0.09(800-t) + 0.12t = 79.50$

$$72 \underset{-72}{} - 0.09t + 0.12t = 79.5 \underset{-72}{}$$

$$\frac{0.03t}{0.03} = \frac{7.5}{0.03}$$

$\boxed{t = 250}$

Step 3 $n + t = 800$ $t = 250$

$$n + 250 \underset{-250}{} = 800 \underset{-250}{}$$

$\boxed{n = 550}$

∴ She invested $550 @ 9%
and $250 @ 12%.

5.   The sum of two numbers is 9. Three times one of the numbers is 15 more than the
     other number. Find the numbers.

     Let x & y be the two numbers   ① $x + y = 9$
                                     ② $3x - y = 15$

Step 1 Rearrange ① to isolate "x"

$x + y = 9$
③ $\boxed{x = 9 - y}$

Step 2 sub $3x - y = 15$

$3(9-y) - y = 15$

$$27 \underset{-27}{} - 3y - y = 15 \underset{-27}{}$$

$$\frac{-4y}{-4} = \frac{-12}{-4}$$

$\boxed{y = 3}$

Step 3 sub $x + y = 9$

$$x + 3 \underset{-3}{} = 9 \underset{-3}{}$$

$\boxed{x = 6}$

∴ $x = 6$
  $y = 3$

6. For the school play, one adult ticket costs $5.00 and one student ticket costs $3.00. Twice as many student tickets as adult tickets were sold. The total <u>revenue</u> was $1650. How many of each kind of ticket were sold?

Let "a" be the number of adult tickets sold
Let "s" be the number of student tickets sold

① Twice as many student tickets as adult tickets $\boxed{S = 2a}$

② The total revenue $\boxed{1650 = 5a + 3s}$

Step1 Sub ① into ②
$1650 = 5a + 3(2a)$
$1650 = 5a + 6a$
$\dfrac{1650}{11} = \dfrac{11a}{11}$
$\boxed{a = 150}$

Step2 Sub
$S = 2a$
$S = 2(150)$
$\boxed{S = 300}$

∴ 300 student, 150 adult tickets were sold.

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7. Martin wants to hire a plumbing company to fix his leaky faucet. Plumbers R Us charges $50 for the initial consultation and $30/hr for any work required. Faucet Fixers charges $30 for the initial consultation and $40/hr for labour. Which company should Martin choose? What assumptions did you make?
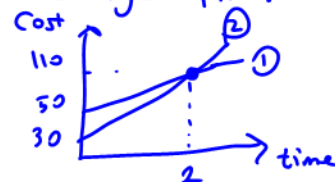
Let "C" be the total cost
Let "h" be the total time

Plumbers R Us ① $C = 30h + 50$
Faucet Fixers ② $C = 40h + 30$

Step1 Sub ① into ②
$\begin{aligned} 30h + 50 &= 40h + 30 \quad {}^{-30} \\ {}^{-30h}\;30h + 20 &= 40h \quad {}^{-30h} \\ \dfrac{20}{10} &= \dfrac{10h}{10} \end{aligned}$
$\boxed{h = 2}$

Step2 $C = 30h + 50$
$= 30(2) + 50$
$= 60 + 50$
$\boxed{C = 110}$

∴ For 2 hours work both companies charge $110.

CONCLUSIONS
① If it is going to take less than 2 hours Faucet Fixers is cheaper
② " " " " " " " 2 hours, both companies charge the same
③ " " " " " " " more than 2 hours Plumbers R Us is cheaper.