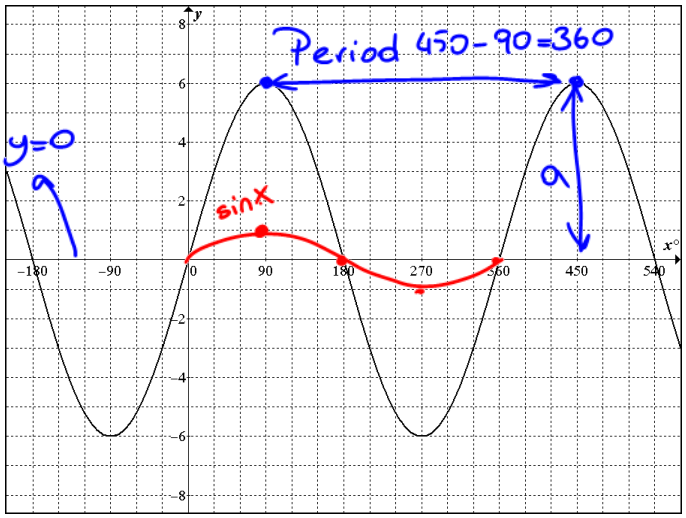
Determining the Equation of a Sinusoidal Function

$$f(x) = a\sin[k(x-d)] + c \text{ and } f(x) = a\cos[k(x-d)] + c$$

c: axis of the curve

Case 1: SINE EQUATION

The function can be considered as a **sine** function. Determine the equation of the function, and then check your answer using desmos. (Note: You need to choose degrees on Desmos. Just click)



1) Determine the key features.

- axis of the curve: $\frac{6+(-6)}{2} = 0$, $c=0$
- amplitude: $\frac{6-(-6)}{2} = 6$, $a=6$
- period = $\frac{360}{k}$: $360 = \frac{360}{k}$, $k=1$

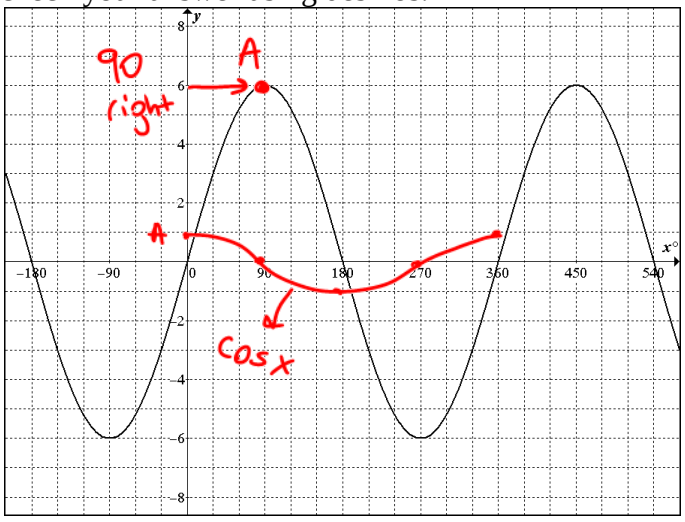
$$y = a\sin[k(x-d)] + c$$

$$y = 6\sin(x-0)$$

$$y = 6\sin(x)$$

Case 2: COSINE EQUATION

The function ~~above~~ can be considered as a **cosine** function. Determine the equation of the function and then check your answer using desmos.

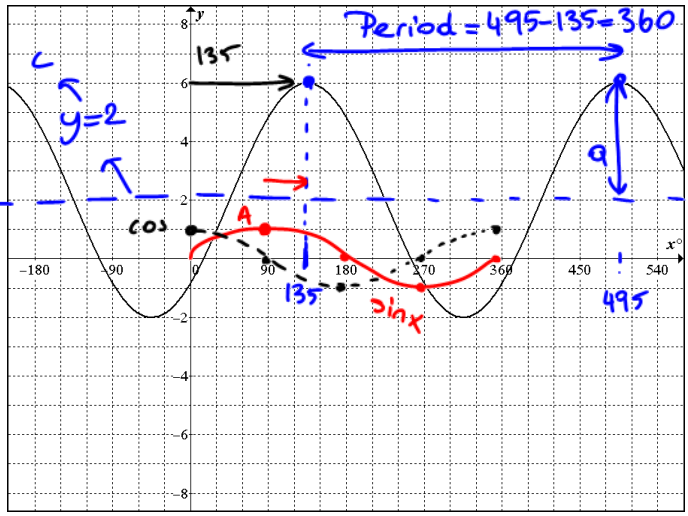


Key features same as above

$c=0$ $a=6$ $k=1$

$$y = 6\cos(x-90)^{\circ}$$

Ex1: Write **two** equations to represent each function.



- axis of the curve $\frac{6+(-2)}{2}=2$ $c=2$
- amplitude $\frac{6-(-2)}{2}=4$ $a=4$
- Period $=\frac{360}{k}$ $360=\frac{360}{k}$ $k=1$
- shift 45 to right for sin, 135 to right for cos

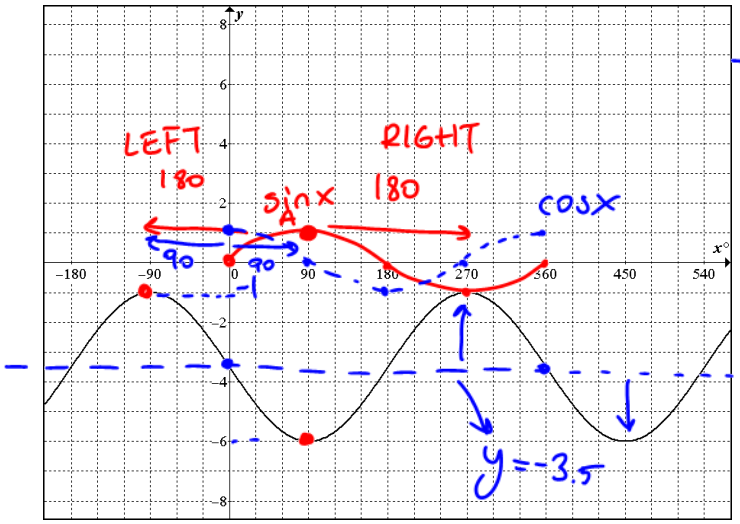
Sine equation $\Rightarrow y=a\sin[k(x-d)]+c$

$y=4\sin(x-45)+2$

cos equation $\Rightarrow y=a\cos[k(x-d)]+c$

$y=4\cos(x-135)+2$

Ex2: Write **two** equations to represent each function.



- axis of the curve $\frac{-1+(-6)}{2}=-3.5$ $c=-3.5$
- amplitude $\frac{-1-(-6)}{2}=2.5$ $a=2.5$
- Period $=\frac{360}{k}$ $360=\frac{360}{k}$ $k=1$

① $y=a\sin[k(x-d)]+c$ $y=-2.5\sin(x)-3.5$

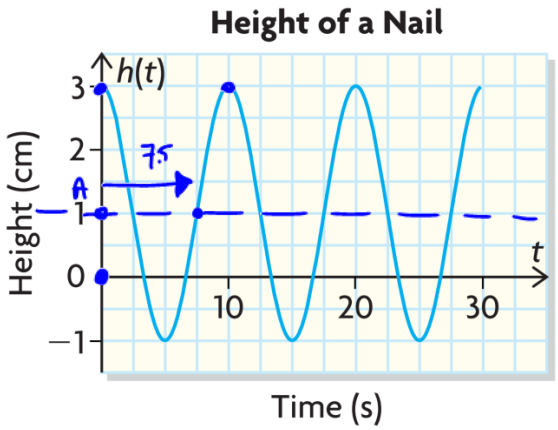
② 180° to LEFT $y=2.5\sin(x+180)-3.5$

③ 180° to RIGHT $y=2.5\sin(x-180)-3.5$

$y=-2.5\cos(x-90)-3.5$ $y=2.5\cos(x+90)-3.5$

$y=2.5\cos(x-270)-3.5$

Ex3: A nail located on the circumference of a water wheel is moving as the current pushes on the wheel. The height of the nail in terms of time can be modeled by the graph shown. **Determine the equation** of a sinusoidal function from its graph.



- axis of the curve $c=\frac{3+(-1)}{2}=1$ $c=1$
- amplitude $a=\frac{3-(-1)}{2}=2$ $a=2$
- Period $=\frac{360}{k}$ $10=\frac{360}{k}$ $k=\frac{360}{10}=36$ $k=36$

Shift $\Rightarrow d=7.5$

$y=a\sin[k(x-d)]+c$ $=2\sin[36(x-7.5)]+1$

or $y=a\cos[k(x-d)]+c$ $y=2\cos[36x]+1$