Pascal's Triangle: Binomial Expansion

This triangular sequence was first discovered by the Chinese mathematician 杨辉 (Yáng Huī, ca. 1238–1298) and was first published by 朱世杰 (Zhū Shìjié) in 1303. However, is it named by Western mathematicians in honour of the French mathematician Blaise Pascal (1623-1662), who discovered it independently at age 13. Following the pattern, fill in row 5 and row 6…

Pascal's Triangle:

Row 0	1
Row 1	1 1
Row 2	1 2 1
Row 3	1 3 3 1
Row 4	1 4 6 4 1
Row 5	1 5 10 10 5 1
Row 6	1 6 15 20 15 6 1

Pascal found many mathematical uses for the array, especially probability theory. We will look at how it applies to binomial expansion.

Binomial	Expansion	Triangle	Row
$(x+y)^0$	1	1	0
$(x+y)^1$	$x+y$	1 1	1
$(x+y)^2$	$x^2+2xy+y^2$	1 2 1	2
$(x+y)^3$	$x^3 + 3x^2y + 3xy^2 + y^3$	1 3 3 1	3
$(x+y)^4$	$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$	1 4 6 4 1	4
$(x+y)^5$	$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$	1 5 10 10 5 1	5
$(x+y)^6$	$1x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$	1 6 15 20 15 6 1	6

1 7 21 35 35 21 7 1

Ex1. $(x-7)^7$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

$1x^7 + 7x^6(-7) + 21x^5(-7)^2 + 35x^4(-7)^3 + 35x^3(-7)^4 + 21x^2(-7)^5 + 7x(-7)^6 + 1(-7)^7$

$x^7 - 49x^6 + 1029x^5 - 12005x^4 + 84035x^3 - 352947x^2 + 823543x - 823543$

Expand each of the following:

a) $(x - 1)^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$1x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$

c) $(2n - 3)^4$

$1(2n)^4 + 4(2n)^3(-3) + 6(2n)^2(-3)^2 + 4(2n)(-3)^3 + 1(-3)^4$

$16n^4 - 96n^3 + 216n^2 - 216n + 81$

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d) $(a + 4b)^3$

$1(a)^3 + 3(a)^2(4b) + 3(a)(4b)^2 + 1(4b)^3$

$a^3 + 12a^2b + 48ab^2 + 64b^3$

e) $(3x - 2y)^5$

$1(3x)^5 - 5(3x)^4(2y) + 10(3x)^3(2y)^2 - 10(3x)^2(2y)^3 + 5(3x)(2y)^4 - 1(2y)^5$

$243x^5 - 810x^4y + 540x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$