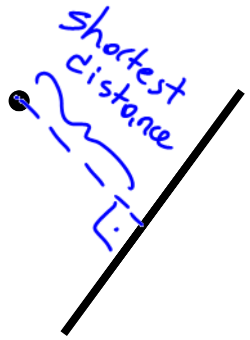
Shortest Distance from a Point to a Line



- Given the point we can draw infinite different lines to the line but…
- The shortest distance is the line that hits it at a 90°
- The shortest distance from a point to a line is the perpendicular distance from the point to your line.

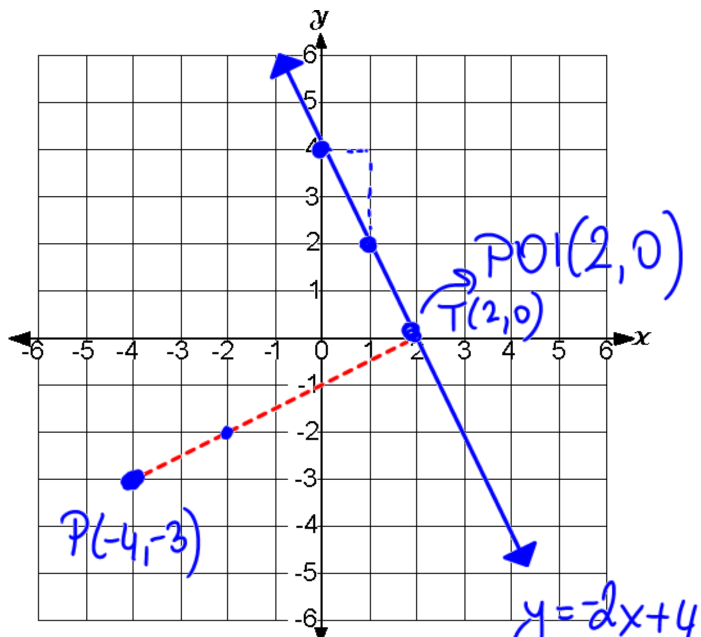
METHOD 1: Finding the Shortest Distance Graphically

Ex1. Find the shortest distance **graphically** between the point P $(-4,-3)$ and the equation $y = -2x + 4$.

Step 1: Graph $y = -2x + 4$

$m = \frac{-2}{1}$ y-int

$$\boxed{m_{\text{shortest distance}} = \frac{1}{2}}$$



$$\overline{PT} = \sqrt{(-4-2)^2 + (-3-0)^2}$$

$$= \sqrt{(-6)^2 + (-3)^2}$$

$(-6)^2 = (-6)(-6) = 36$

$(-3)^2 = (-3)(-3) = 9$

$$= \sqrt{36 + 9}$$

$$= \sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

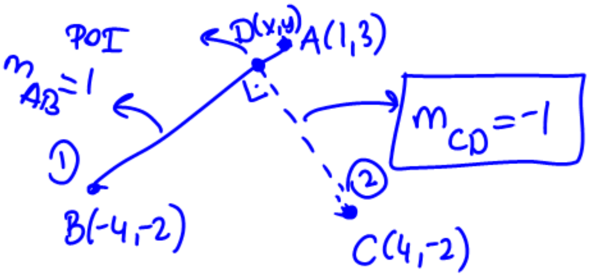
perfect square

exact form

METHOD 2: Finding the Shortest Distance Algebraically

Ex2. Find the shortest distance from the point $C(4,-2)$ to the line passing through the points $A(1,3)$ *and* $B(-4,-2)$.

Step 1 Find equation of the line AB.
Step 2 Draw a line perpendicular to AB that goes through C. Let the point on AB be called D. Find the equation of the line CD.
Step 3 Find D, the POI of AB and CD (substitution or elimination).
Step 4 Find the length of CD.



Step 1: Equation $\overline{AB}$

$A(1,3)$ $B(-4,-2)$

$m_{AB} = \frac{3-(-2)}{1-(-4)} = \frac{3+2}{1+4} = \frac{5}{5}$

$\boxed{m_{AB}=1}$ $A(1,3)$

$y = m(x-p)+q$

$y = 1(x-1)+3$

$y = x-1+3$

① $\boxed{y = x+2}$

Step 2: Equation of $\overline{CD}$

$m_{CD}=-1$ $C(4,-2)$

$y = m(x-p)+q$

$y = -1(x-4)+(-2)$

$y = -x+4-2$

② $\boxed{y=-x+2}$

Step 3: Find POI D(x,y)

① $y = x+2$

② $y = -x+2$

Sub ① → ②

$x+2 = -x+2$

$x+x = 2-2$

$\frac{2x}{2} = \frac{0}{2}$

$\boxed{x=0}$

$y = x+2$

$y = 0+2$

$\boxed{y=2}$

POI ⇒ D(0,2)

Step 4: Shortest distance

$C(4,-2)$ $D(0,2)$

$\overline{CD} = \sqrt{(4-0)^2+(-2-2)^2}$

$= \sqrt{(4)^2+(-4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= \sqrt{16 \cdot 2}$

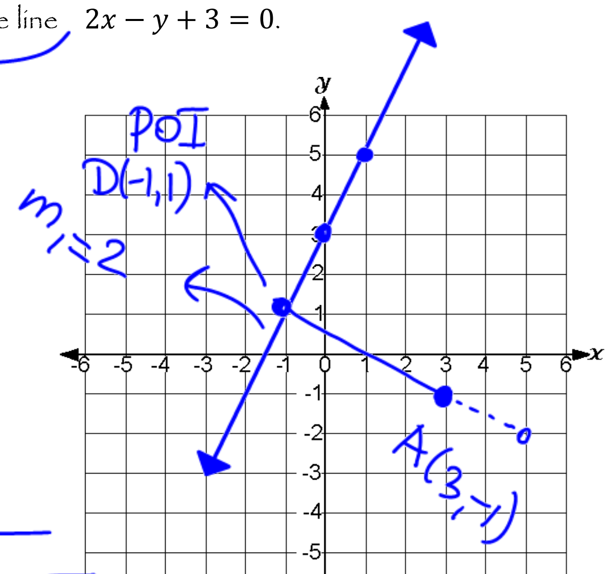
$= \sqrt{16}\cdot\sqrt{2}$

$= 4\sqrt{2}$

∴ The shortest distance from C to $\overline{AB}$ is $\sqrt{32}$ or $4\sqrt{2}$

Practice

Ex3. Determine the shortest distance **graphically** from $A(3,-1)$ to the line $2x - y + 3 = 0$.



Step1: Rearrange $2x - y + 3 = 0$

$2x + 3 = y$

$y = 2x + 3$

$m = 2$ y-int

Step2: Slope of shortest distance

$m = \frac{-1}{2}$

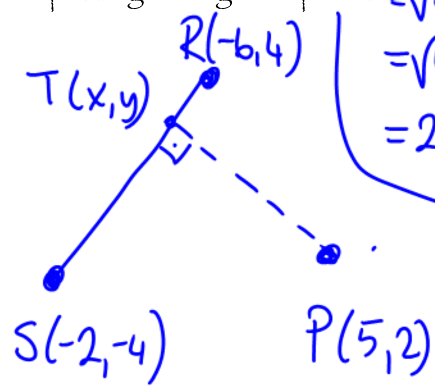
Step3: $\overline{AD} = \sqrt{(-1-3)^2 + (1-(-1))^2} = \sqrt{(-4)^2 + (2)^2} = \sqrt{16+4} = \sqrt{20}$

$= \sqrt{4 \cdot 5}$

$= \sqrt{4}\sqrt{5}$

$= 2\sqrt{5}$

Ex4: **Algebraically** determine the shortest distance from the point $P(5,2)$ to the line passing through the points $R(-6,4)$ *and* $S(-2,-4)$.



Step1: Equation of $\overline{RS}$

$m_{RS} = \frac{-4-4}{-2-(-6)} = \frac{-8}{-2+6} = \frac{-8}{4} = -2$

$\boxed{m_{RS} = -2}$

$m = -2$ S(-2,-4) T(x,y)

$y = m(x-p) + q$

$y = -2[x-(-2)] + (-4)$

$y = -2(x+2) - 4$

$y = -2x - 4 - 4$

① $\boxed{y = -2x - 8}$

Step2: Equation of PT

$m_{PT} = \frac{1}{2}$ P(5,2)

$y = m(x-p) + q$

$y = 0.5(x-5) + 2$

$y = 0.5x - 2.5 + 2$

② $\boxed{y = 0.5x - 0.5}$

Step3: Finding POI

Sub ① → ②

$-2x - 8 = 0.5x - 0.5$

$-8 + 0.5 = 0.5x + 2x$

$\frac{-7.5}{2.5} = \frac{2.5x}{2.5}$

$\boxed{x = -3}$

$y = -2(-3) - 8$

$y = 6 - 8$

$\boxed{y = -2}$

Step4: Finding s.d

$\overline{PT} = \sqrt{(-3-5)^2 + (-2-2)^2}$

$= \sqrt{64 + 16}$

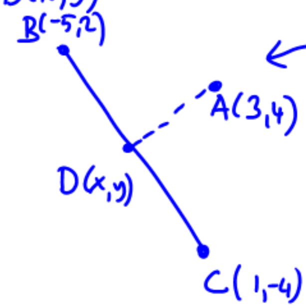
$= \sqrt{80}$

$= 4\sqrt{5}$

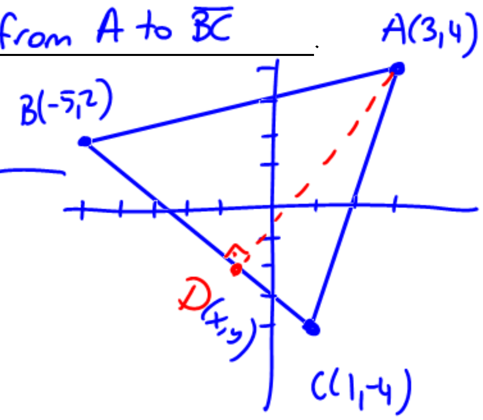
∴ The shortest distance $4\sqrt{5}$

Ex 5. Triangle ABC has vertices $A(3,4)$, $B(-5,2)$, and $C(1,-4)$. Determine an equation for AE, the **altitude** from A to BC. What is the area of triangle ABC?

An altitude of a triangle is the shortest distance from A to $\overline{BC}$.



We need to create a linear system so that we can find the POI which is D(x,y)



Step1: Determine the equation of $\overleftrightarrow{BC}$

$m_{BC} = \frac{-4-2}{1-(-5)} = \frac{-6}{6} = -1$ $\boxed{m_{BC} = -1}$

$y = m(x-p)+q$ $C(1,-4)$

$= -1(x-1)-4$

$= -x+1-4$

① $\boxed{y = -x-3}$

Step2: Determine the equation of $\overline{AD}$

$m_{AD} = 1$ $A(3,4)$

$y = m(x-p)+q$

$= 1(x-3)+4$

② $\boxed{y = x+1}$

Step3: Find POI

Sub ① into ②

$-x-3 = x+1$

$-4 = 2x$

$\boxed{-2 = x}$

$y = x+1$

$= -2+1$

$\boxed{y = -1}$

$D(-2,-1)$

Step4: Calculate the length of Shortest distance

A(3,4) and D(-2,-1)

$d = \sqrt{(-2-3)^2+(-1-4)^2}$

$= \sqrt{25+25}$

$= \sqrt{50}$

$= 5\sqrt{2}$

B(-5,2) C(1,-4)

Step5: Calculate the length of $\overleftrightarrow{BC}$

$d = \sqrt{(-5-1)^2+(2-(-4))^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$

$= 6\sqrt{2}$

Step6: $\text{Area} = \frac{b \cdot h}{2} = \frac{6\sqrt{2} \cdot 5\sqrt{2}}{2}$

$= 30$

∴ The area is 30