Investigating How to Factor a Simple Trinomial

Part A: Using FOIL

Expand and simplify each of the following.

Question ↓ Answer ↓

a. $(x+3)(x+2) = (x)(x)+(x)(2)+(3)(x)+(3)(2)$
$= x^2+2x+3x+6$
$= x^2+5x+6$

b. $(x-4)(x-1) = x^2-x-4x+4$
$= x^2-5x+4$

c. $(x+4)(x-3) = x^2-3x+4x-12$
$= x^2+x-12$

d. $(x-2)(x+5) = x^2+5x-2x-10$
$= x^2+3x-10$

e. $(x-6)(x+2) = x^2+2x-6x-12$
$= x^2-4x-12$

f. $(x-3)(x-7) = x^2-7x-3x+21$
$= x^2-10x+21$

Part B: Making the Connection

Consider the question being in the form $(x+m)(x+n)$, and the answer being in the form x^2+bx+c. Complete the chart based on the 8 questions above. The first question has been completed for you.

	Question			Answer		
	$(x+m)(x+n)$	m	n	x^2+bx+c	b	c
a.	$(x+3)(x+2)$	3	2	x^2+5x+6	5	6
b.	$(x-4)(x-1)$	-4	-1	x^2-5x+4	-5	4
c.	$(x+4)(x-3)$	4	-3	x^2+x-12	1	-12
d.	$(x-2)(x+5)$	-2	5	$x^2+3x-10$	3	-10
e.	$(x-6)(x+2)$	-6	2	$x^2-4x-12$	-4	-12
f.	$(x-3)(x-7)$	-3	-7	$x^2-10x+21$	-10	21

1. What is the relationship between **m, n, and b**?

   $b = m + n$

2. What is the relationship between **m, n, and c**?

   $c = m \cdot n$

3. Try expanding $(x + 6)(x + 3)$ without FOILing (i.e. use what you discovered above about the relationship of the numbers to skip right to the answer).

   $(x + 6)(x + 3) = x^2 + \underline{9}\,x + \underline{18}$

4. Try expanding $(x - 5)(x + 3)$ without FOILing (i.e. use what you discovered above about the relationship of the numbers to skip right to the answer).

   $(x - 5)(x + 3) = x^2 \underline{-2}\,x \underline{-15}$

Part C: Doing Some Practice

Fill in the blanks. Do not FOIL!!

a. $(x + 5)(x + 1) = x^2 + \underline{6}\,x + \underline{5}$

b. $(x - 2)(x - 6) = x^2 - \underline{8}\,x + \underline{12}$

c. $(x - 3)(x + 2) = x^2 - \underline{1}\,x - \underline{6}$

d. $(x + 8)(x - 2) = x^2 + \underline{6}\,x - \underline{16}$

e. $(x + 5)(x + \underline{2}) = x^2 + 7x + 10$

f. $(x - \underline{3})(x - 9) = x^2 - 12x + 27$

g. $(x - \underline{6})(x + 3) = x^2 - 3x - 18$

h. $(x + 12)(x - \underline{1}) = x^2 + 11x - 12$

i. $(x + 4)(x + \underline{2}) = x^2 + 6x + \underline{8}$

j. $(x - \underline{2})(x - 5) = x^2 - \underline{7x} + 10$

k. $(x + \_3\_)(x - 1) = x^2 + \_2\_x - 3)$

l. $(x - 4)(x + \_1\_) = x^2 - 3x - \_4\_$

m. $(x + \_3\_)(x + \_4\_) = x^2 + 7x + 12$

n. $(x - \_2\_)(x - \_10\_) = x^2 - 12x + 20$

o. $(x + \_\_\_)(x - \_\_\_) = x^2 + 4x - 2$

p. $(x - \_5\_)(x + \_4\_) = x^2 - x - 20$

Guess what? If you could answer questions m, n, o, & p, then you can factor a simple trinomial! When asked to 'Factor', your Question and Answer are switched.

Example: Factor

Question ↓	*Answer* ↓
a. $x^2 + 8x + 12$	$= (x + \_2\_)(x + \_6\_)$ (2 & 6)
b. $x^2 - 7x + 10$	$= (x-2)(x-5)$

Explain, in your own words, how to get from the question to the answer when factoring simple trinomials.

Find Two numbers that multiply to the last term which add up to the coefficient of the middle term...

Part D: The Hardest Part . . .

The hardest part of factoring simple trinomials is often finding the two numbers, **m** and **n**. These are the thoughts you need to take to help.

Consider this example: $x^2 + 5x - 24$. Let's pretend that you can't find the two numbers that multiply to -24 and add to +5. Answer these questions.

1. Write down ALL of the pairs of numbers that multiply to 24 in the table. All of the rows here should be used.

2. Remember that they have to multiply to a NEGATIVE number. What does this mean about the signs (i.e. + or -) of **m** and **n**?

   Either m or n is "-"

possible m and n values	
m	**n**
1	24
2	12
3	8
4	6

3. Now you have to decide which one is positive and which one is negative. Look at the middle term. **m** and **n** need to ADD to a POSITIVE number. What does this tell you about which of **m** or **n** is positive and which is negative?

   The bigger number must be positive

4. So . . what is **m** = __+8____ and **n** = __-3____

Consider this example: $x^2 - 9x + 18$. Let's pretend that you can't find the two numbers that multiply to 24 and add to -9. Answer these questions.

5. Write down ALL of the pairs of numbers that multiply to 18 in the table. All of the rows here should be used.

6. Remember that they have to multiply to a POSITIVE number. What does this mean about the signs (i.e. + or -) of **m** and **n**?

m and n both must be either "-" or "+"

possible m and n values	
m	n
1	18
2	9
3	6

7. If they need to ADD to a NEGATIVE number, then will both **m** and **n** be positive or both be negative?

negative

8. So . . what is **m** = -3 and **n** = -6

Part E: Practice

9. Factor, if possible. If it is not possible, state why not.

a. $x^2 + 12x + 27 = (x+3)(x+9)$

b. $x^2 - 8x + 12 = (x-2)(x-6)$

c. $x^2 + x - 12 = (x+3)(x-4)$

d. $x^2 - 3x - 10 = (x+2)(x-5)$

e. $3x^2 - 18x + 27 = 3(x^2 - 6x + 9)$
common factor the 3 first
$= 3(x-3)(x-3)$
factor the remaining simple trinomial

f. $2x^2 + 10x + 12 = 2(x^2 + 5x + 6)$
common factor the 2 first
$= 2(x+2)(x+3)$
factor the remaining simple trinomial