'Intersecting' Lines!?!
- Things that Make You Go Hmmmm…

Warm-up: Ted and Ned are going to race their dirt bikes. Since Ted is younger, Ned is going to give him a 10 mile head start. Ted travels at 10 mph and Ned travels at 20 mph.
At what time will Ned catch up with Ted?
How far will they have traveled when they meet?

Ted's equation: $y = 10x + 10$ x = time in hours
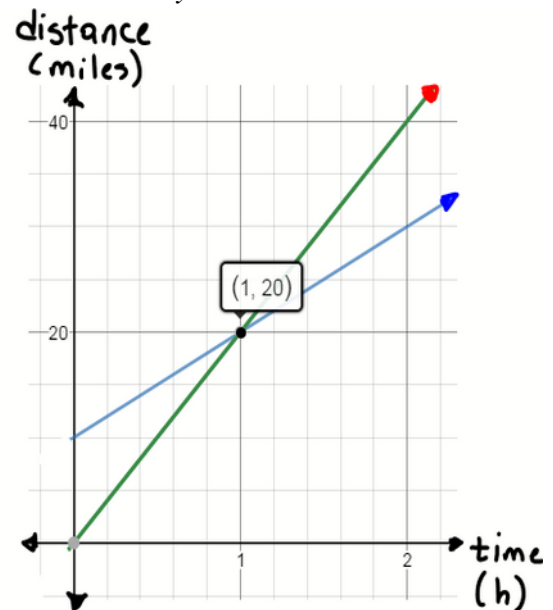**Ned's** equation: $y = 20x + 0$ y = distance in miles

Turn on your **DESMOS**

1. Use the Online Graphing Calculator to graph the two lines. **Sketch** them on the grid to the right. Be sure the point of intersection is showing.

2. Determine the point of intersection of the two lines.

   ____(1,20)____

3. What does this point represent in the context of this word problem?

   It's when Ned catches up with Ted

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Do 2 lines always intersect in one point? Check it!

Yes, zoom in or out two lines only intersect in one point.

Task 1: One Solution

- Click/ touch on the "**x**" to delete the equations.
- Change the equation to $y = -2x + 6$, and then change the colour of the line to black.
- Change the equation to y = 4x -5, then change the colour of the line to orange.

4. Sketch the two graphs on the grid provided.
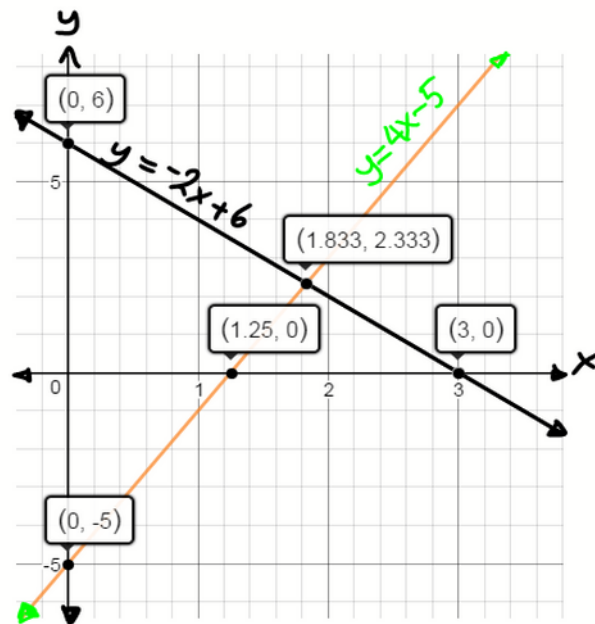
5. Why is there one solution to the linear system $\boxed{\begin{array}{l} y = -2x+6 \\ y = 4x - 5 \end{array}}$?

B/c there is only one intersection point.

6. How can you tell by looking at the equations that there will be one solution to the linear system?
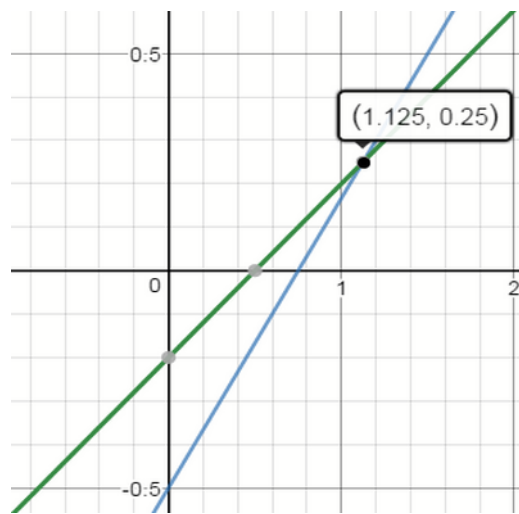**They both have different slopes**

7. Predict the equation of another line which would have one solution with $y = -2x + 6$.

$y = 5x - 6$ _____ Verify your answer by graphing it on the Desmos.

- Using the **green** line, change the equation to:
  $2x - 5y = 1$
- Using the **blue** line, change the equation to:
  $4x - 6y = 3$

8. Sketch the two graphs on the grid provided.

9. Why is there one solution to the linear system ① $\boxed{\begin{array}{l} 2x - 5y = 1 \\ 4x - 6y = 3 \end{array}}$?

B/c there is only one intersection point.

10. How can you tell by looking at the equations that there will be one solution to the linear system?

If the first ratios are different, ONE SOLUTION. $\dfrac{2}{4} \neq \dfrac{-5}{-6} \neq \dfrac{1}{3}$

11. Predict the equation of another line which would have one solution with $2x - 5y = 1$.

$3x - 10y = 5$ _____ Verify your answer by graphing it on the Desmos.
$6x + 10y = 2$

$\dfrac{2}{a} \neq \dfrac{-5}{b} \neq \dfrac{1}{c}$
↓ ↓ ↓
6 10 2

$2x - 5y = 1$

$ax + by = c$
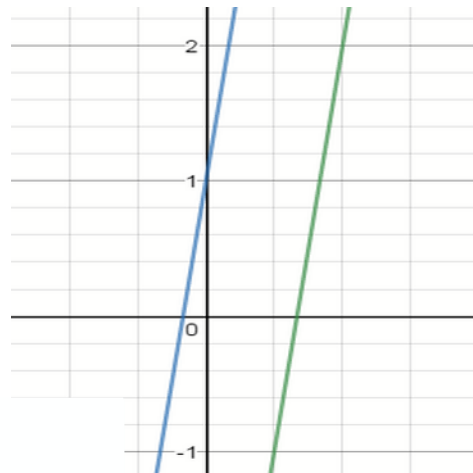
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Task 2: No Solution

- Using the **green** line, change the equation to: $y = 3x - 4$
- Using the **blue** line, change the equation to: $y = 3x + 1$

12. Sketch the two graphs on the grid provided.

13. Why is there no solution to the linear system $\begin{array}{l} y = 3x - 4 \\ y = 3x + 1 \end{array}$?

   Because the lines do not intersect.

14. How can you tell by looking at the equations that there will not be a solution to the linear system?
   They both have the same slope and different y-intercepts.

15. Predict the equation of another line which would have no solution with $y = 3x - 4$.

   y = 3x - 1
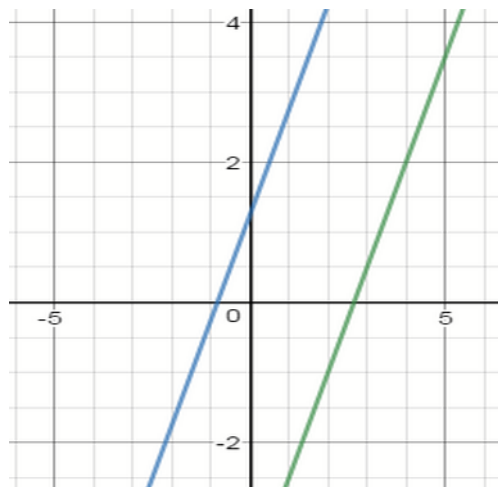   _____ Verify your answer by graphing it on the Desmos.

- Using the **green** line, change the equation to:
   $3x - 2y = 8$
- Using the **blue** line, change the equation to:
   $6x - 4y = -5$

16. Sketch the two graphs on the grid provided.

17. Why is there no solution to the linear system $\begin{array}{l} 3x - 2y = 8 \\ 6x - 4y = -5 \end{array}$?

   Because two lines are parallel and do not intersect.

18. How can you tell by looking at the equations that there will not be a solution to the linear system?
   If the first two Ratios are same but third one is different it is a NO SOLUTION $\dfrac{3}{6}, \dfrac{-2}{-4}, \dfrac{8}{-5} \Rightarrow \dfrac{1}{2} = \dfrac{1}{2} \ne \dfrac{-8}{5}$

19. Predict the equation of another line which would have no solution with $3x - 2y = 8$.

   15x - 10y = 4
   _____ Verify your answer by graphing it on the Desmos.

   ① $3x - 2y = 8$
   ② $15x - 10y = 4$

   ① $3x - 2y = 8$
   ② $9x - 6y = -8$

   $\dfrac{3}{9}, \dfrac{-2}{-6}, \dfrac{8}{-8} \Rightarrow \dfrac{1}{3} = \dfrac{1}{3} \ne -1$

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Task 3: Many Solutions

- Using the **green** line, change the equation to:
  $2x - 3y = 4$
- Using the **blue** line, change the equation to:
  $4x - 6y = 8$

20. Sketch the two graphs on the grid provided.
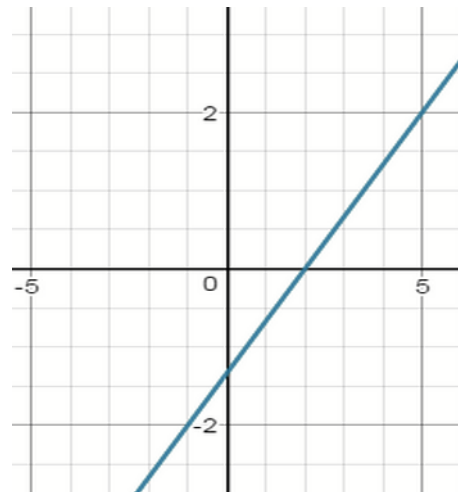
21. Why are there multiple solutions to the linear system $\begin{aligned} 2x - 3y &= 4 \\ 4x - 6y &= 8 \end{aligned}$?

Because two lines are coincident. They sit on top of each other.

22. How can you tell by looking at the equations that there will be multiple solutions to the linear system?

If all of the ratios are the same then it is an Infinite (multiple) solution

$$\frac{2}{4}, \frac{-3}{-6}, \frac{4}{8} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$$

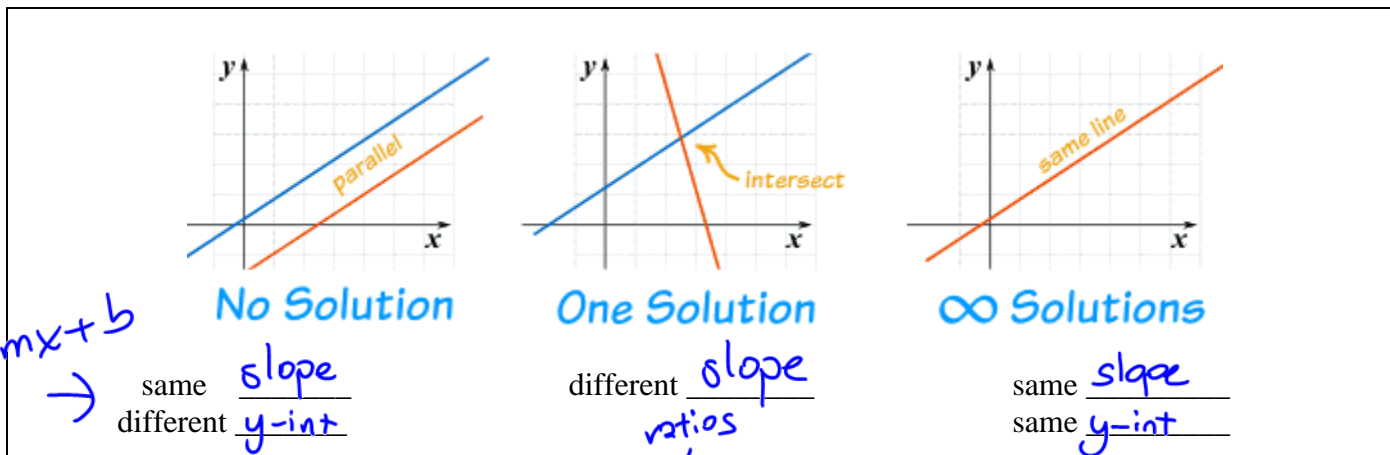23. Predict the equation of another line which would have multiple solutions with $2x - 3y = 4$.

   $6x - 9y = 12$
   _____ Verify your answer by graphing it on the Desmos.

SUMMARY

$y = mx + b$

No Solution
same __slope__
different __y-int__

Only the first two ratios are the same

STD form

One Solution
different __slope__
__ratios__

The first two are different

∞ Solutions
same __slope__
same __y-int__

The ratios are all the Same

24. Determine the number of solutions each linear system has. Justify your decision.

a. $3x - y = 5$
   $2x + 3y = 6$
   ONE ✗

b. $\times 3 \begin{cases} 3x + 4y = 12 \\ -9x - 12y = -36 \end{cases} \times 3$
   infinite ✓

c. $y = 3x - 5$
   $y = 4x + 6$
   ONE ✗

d. $\times 5 \begin{cases} 2x - 3y = 10 \\ -10x + 15y = -15 \end{cases}$
   NONE //

e. $\times 0.5 \begin{cases} x + 2y = 10 \\ 0.5x + y = 8 \end{cases}$
   NONE //

f. $3x - 5y - 2 = 0$
   $4x + 5y + 2 = 0$
   ONE

g. $y = 4x - 3$
   $y = 4x - 7$
   NONE

h. $x + y = 0$
   $x - y = 0$
   ONE

25. Sarah and Shannon mow lawns during the summer to earn money. They both calculated their start-up expenses, operating expenses, and income per hour of mowing. They wrote these equations for their income, *I*, after *h* hours of mowing.

    $I = 10.25h - 125$ *Sarah*

    $I = 10.25h - 100$ *Shannon*

    a.   What are Shannon's start-up costs? $100

    b.   What does Sarah charge per hour? $10.25

    c.   Will Sarah ever earn as much money as Shannon? Justify your decision.
    Depending on how many hours Sarah might get. If Sarah gets as many hours as Shannon will, then she will make less. On the other hand if
    d.   Sketch what these two graphs would look like. Sarah gets more hours, she might earn as much money as Shannon.
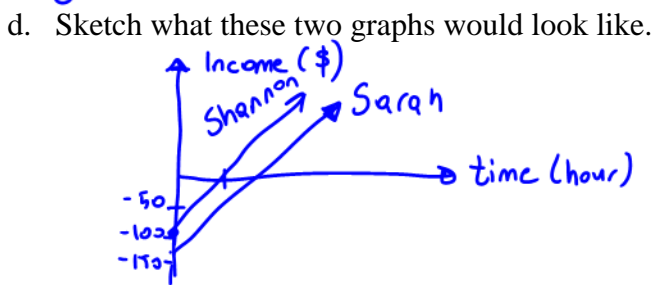
26. An air traffic controller is plotting the course of two jets scheduled to land in about 15 minutes. One aircraft is following a path defined by the equation $3x - 5y = 20$ and the other by the equation $18x = 30y + 72$. Should the controller alter the paths of either aircraft? Justify your decision.

rearrange

① $3x - 5y = 20$
② $18x - 30y = 72$

Two equations are parallel b/c A and B values in the ② equation are 6 times those in the ① equation and C values are different.

OR

Rearrange both equation in $y = mx + b$ form

① $3x - 5y \overset{-3x}{=} 20 \overset{-3x}{}$
   $\dfrac{-5y}{-5} = \dfrac{-3x}{-5} + \dfrac{20}{-5}$
   ① $\boxed{y = \frac{3}{5}x - 4}$

② $8x \overset{-72}{=} 30y + 72 \overset{-72}{}$
   $\dfrac{18x}{30} - \dfrac{72}{30} = \dfrac{30y}{30}$
   ② $\boxed{y = \frac{3}{5}x - \frac{12}{5}}$

∴ They have the same slope and different y-int; therefore, two equations are parallel.

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