Common Factoring

Warm-Up: The area of a park is given by the expression $16x^2 - 4x$. If the length is given by the expression $4x$, what is an expression for the width?

Recall: $A = L \times W$

$$\frac{16x^2}{4x} - \frac{4x}{4x} = \frac{4x \cdot W}{4x}$$

$4x - 1 = W$ ∴ width is $4x-1$

Greatest Common Factor (GCF)

- product of

(1) greatest integer that divides both coefficients

(2) common variable(s) with the lowest exponent

Example: Consider: $16x^3y^5z$ and $24x^6y^4$ The GCF is $8x^3y^4$

To Common Factor:

- determine the GCF
- DIVIDE the polynomial by the GCF (gives the quotient)
- write as a product: $GCF\left(\frac{1^{st}\text{ term}}{GCF} + \frac{2^{nd}\text{ term}}{GCF} + \frac{3^{rd}\text{ term}}{GCF}\right)$

Example 1: Factor $15x^2 + 10x$ (GCF = $5x$)

$= 5x\left(\frac{15x^2}{5x} + \frac{10x}{5x}\right) \Rightarrow 5x(3x^{2-1} + 2x^{1-1}) \Rightarrow 5x(3x+2)$ (x^0)

Example 2: Factor $-16x^4 - 24x^3 + 8x^2$ (GCF = $-8x^2$)

$= -8x^2\left(\frac{-16x^4}{-8x^2} - \frac{24x^3}{-8x^2} + \frac{8x^2}{-8x^2}\right) = 8x^2(2x^{4-2} + 3x^{3-2} - 1 \cdot x^{2-2}) = 8x(2x^2+3x-1)$

Example 3: Factor $9x^2 + 6x + 15$ (GCF = 3)

$= 3\left(\frac{9x^2}{3} + \frac{6x}{3} + \frac{15}{3}\right)$

$= 3(3x^2+2x+5)$

Common Factoring by Grouping

- If there are 4 terms, you may be able to common factor them in pairs

Example 1: Factor $4x^3 + 20x^2 + 5x + 25$

GCF: $4x^2$ GCF: 5

$= 4x^2\left(\frac{4x^3}{4x^2} + \frac{20x^2}{4x^2}\right) + 5\left(\frac{5x}{5} + \frac{25}{5}\right)$

$= 4x^2(x+5) + 5(x+5)$

$= (x+5)(4x^2+5)$

Common factor the first 2 and the second 2 terms in pairs

The 2 binomials in brackets should be equal!

Common factor the binomial in brackets

Example 2: Factor $14m^3 - 21m^2 + 12m - 18$

GCF: $7m^2$ GCF: 6

$= 7m^2\left(\frac{14m^3}{7m^2} - \frac{21m^2}{7m^2}\right) + 6\left(\frac{12m}{6} - \frac{18}{6}\right)$

$= 7m^2(2m-3) + 6(2m-3)$

$= (2m-3)(7m^2+6)$

Example 3: Factor $2k^3 - 7k^2 - 4k + 14$

$= k^2\left(\frac{2k^3}{k^2} - \frac{7k^2}{k^2}\right) - 2\left(\frac{-4k}{-2} + \frac{14}{-2}\right)$

$= k^2(2k-7) - 2(2k-7)$

$= (2k-7)(k^2-2)$

Example 4: Factor $14v^3 - 2v^2 + 7v - 1$

$= 2v^2\left(\frac{14v^3}{2v^2} - \frac{2v^2}{2v^2}\right) + (7v-1)$

$= 2v^2(7v-1) + (7v-1)$

$= (7v-1)(2v^2+1)$