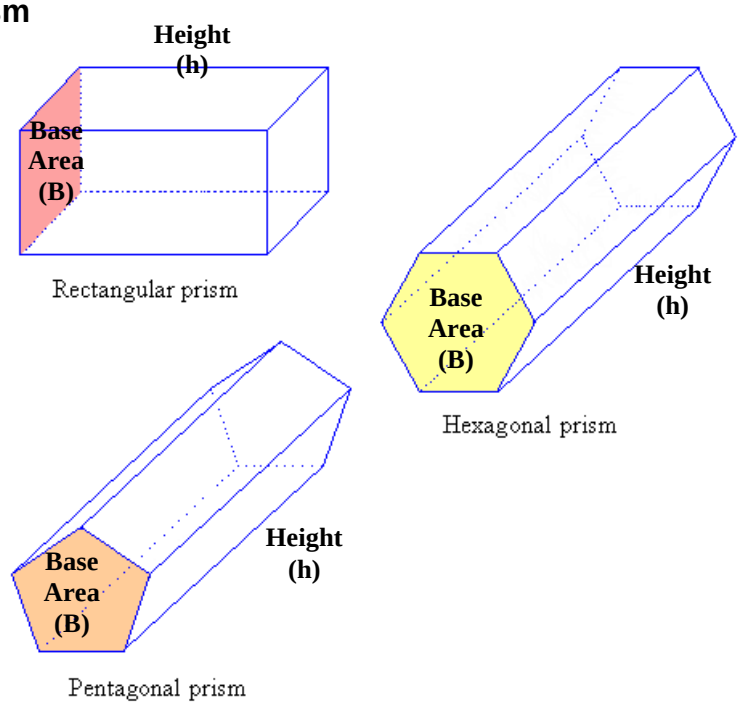
Volume of Prisms & Cylinder

A **prism** is a 3D shape with two identical parallel bases (top and bottom are the same). All other faces are rectangles.

To find the volume of ANY prism, find the area of the base and multiply it by the height.

Any Prism



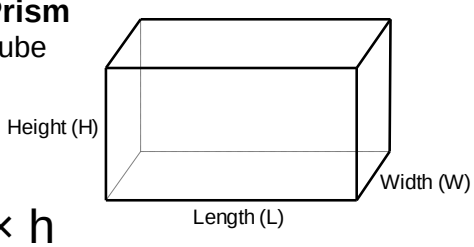
This formula is for ANY PRISM:

$$V = B \times h$$

B is the area of the base (shaded region on the diagrams)

Rectangular Prism

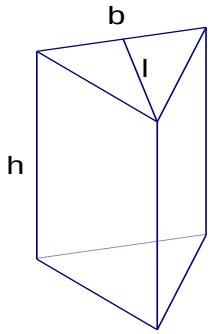
- also includes cube



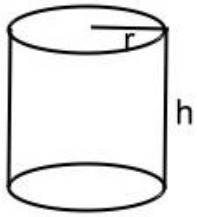
$V = l \times w \times h$

Triangular Prism

$$V = \frac{b \times l \times h}{2}$$



Cylinder – Basically, a circle-based prism

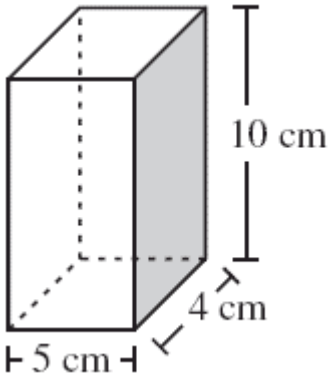


$$V = \pi r^2 h$$

Remember: π = 3.14 (or, there is a π button on your calculator)

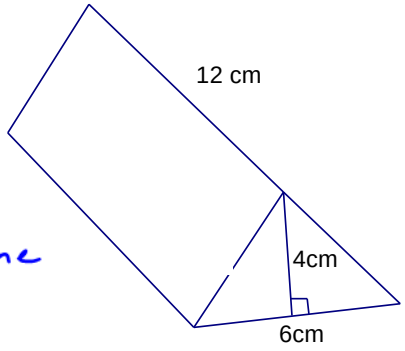
Example 1: Determine the volume of this prism

$V = B \cdot h$
$= 5 \cdot 4 \cdot 10$
$= 200$
∴ The volume is $200\ cm^3$



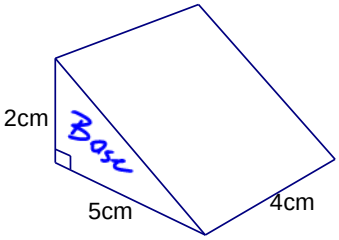
Example 2: Determine the volume of this prism in cm^3.

$V = B \cdot h$
$= \frac{6 \cdot 4^2}{2} \cdot 12$
$= 144$
∴ The volume is $144\ cm^3$

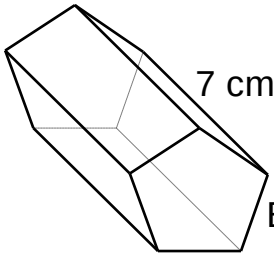


Example 3: Determine the volume of this prism in cm^3.

$V = B \cdot h$
$= \frac{5 \cdot 2}{2} \cdot 4$
$= 20$
∴ The volume is $20\ cm^3$



Example 4: Determine the volume of this prism in cm^3.

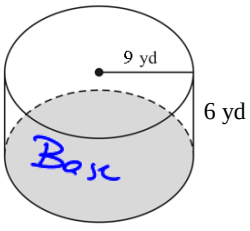


Base Area = $8\ cm^2$

$V = B \cdot h$
$= 8 \cdot 7$
$= 56$
∴ The volume is $56\ cm^3$

Example 5: Determine the volume of this cylinder in yd^3.

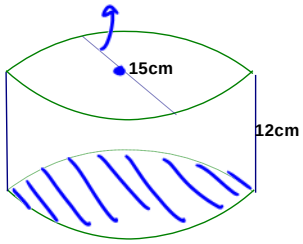
$V = B \cdot h$
$= \pi r^2 \cdot h$
$= \pi (9)^2 \cdot 6$
$= 1526.81$
∴ The volume is $1526.81\ yd^3$



Example 6: Determine the volume of this cylinder in cm^3.

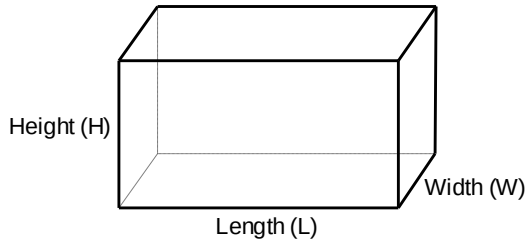
$r = \frac{15}{2} = 7.5$

$V = Bh$
$= \pi (7.5)^2 12$
$= 2120.58$
∴ The volume is $2120.58\ cm^3$



Surface Area of Prisms

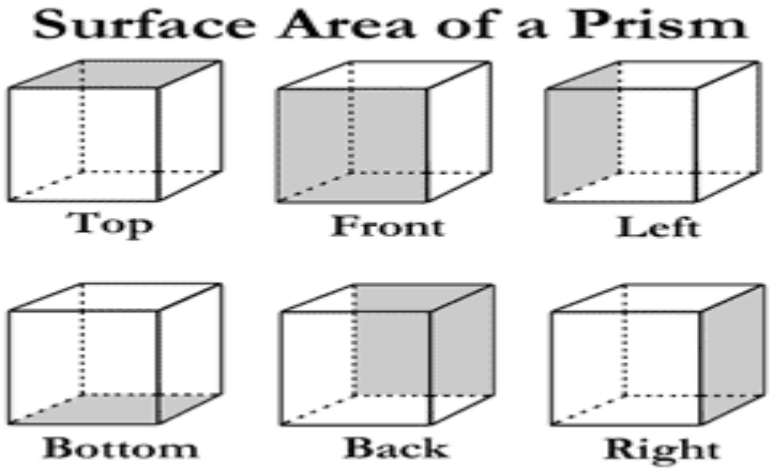
Prism (Rectangular/Right



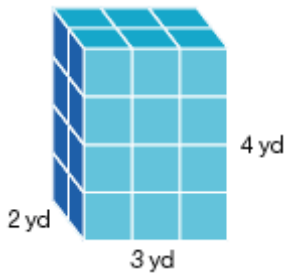
Add the area of all the faces

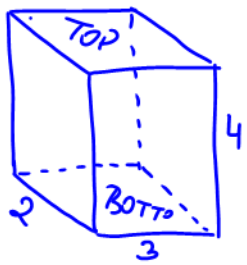
$A = 2(top + front + side)$

$A = 2(lw + lh + wh)$



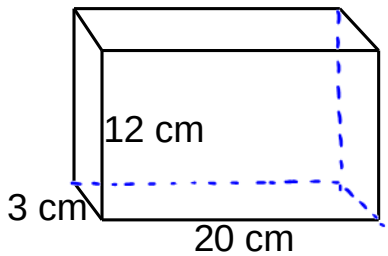
Example 1: Determine the surface area of this prism in yd^2.





$A = 2(2 \cdot 3 + 3 \cdot 4 + 2 \cdot 4)$
$= 2(6 + 12 + 8)$
$= 2(26)$
$= 52$
∴ The SA is $52 yd^2$

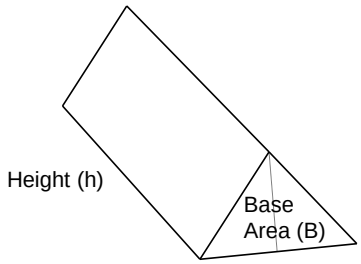
Example 2: Determine the surface area of this box which has **NO LID.**



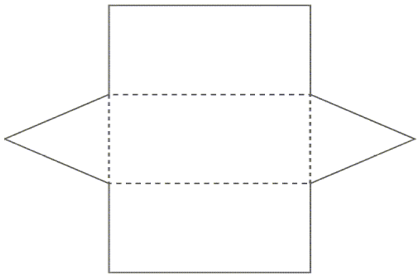
$SA = 2(front + side) + bottom$
$= 2(12 \cdot 24 + 3 \cdot 12) + 3 \cdot 20$
$= 708$
∴ The SA is $708 cm^2$

Any Other Prism (e.g. Triangluar)

Add the area of all the faces

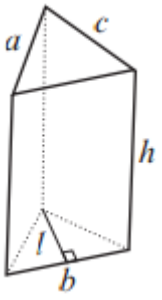


Each side will be a rectangle, Use the Area of a Rectangle formula: $A = L \times W$



In this case, the Base of the prism is a triangle. Use the Area of a Triangle formula:

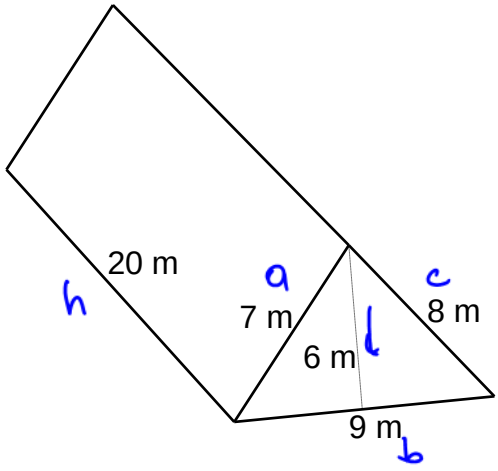
$$A = \frac{bh}{2}$$



If the Base is not a triangle, the area of the base will be given.

$A_{total} = A_{rectangles} + 2A_{base}$
$= ah + bh + ch + bl$

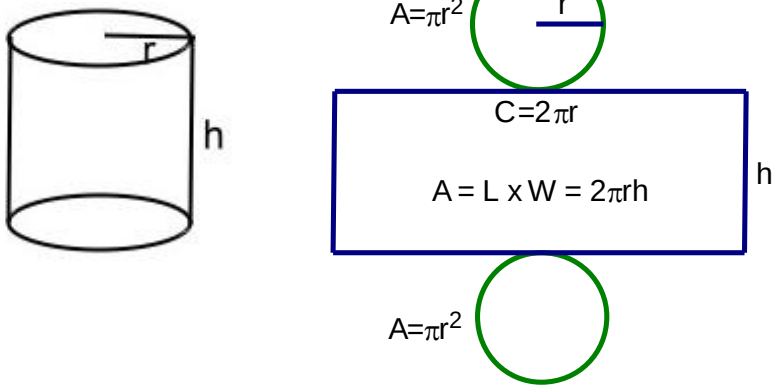
Example 3: Determine the surface area of this prism in m^2.



$SA = ah + bh + ch + bl$
$= 7 \cdot 20 + 9 \cdot 20 + 8 \cdot 20 + 9 \cdot 6$
$= 534$
$\therefore$ SA is $534\ m^2$

Surface Area of Cylinders

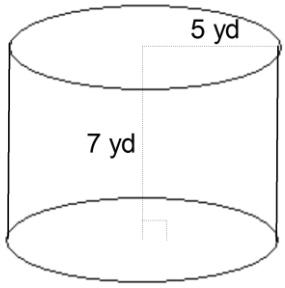
Cylinder (Basically – a Circle Based Prism)



A = 2(top) + side
The top/bottom is a circle

$$A = 2(\pi r^2) + 2\pi rh$$

Example 1: Determine the surface area of this cylinder in yd^2.



$SA = 2(top) + side$

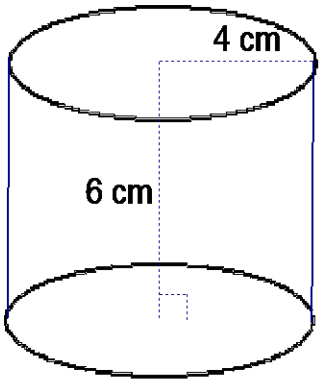
$= 2(\pi r^2) + 2\pi rh$

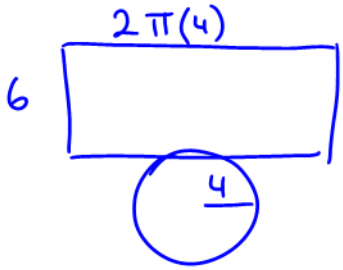
$= 2(\pi 5^2) + 2\pi 5 \cdot 7$

$= 376.99$

$\therefore$ The SA is $376.99\ yd^2$

Example 2: Determine the surface area of this drinking glass.





$SA =$ bottom + side

$= \pi(4)^2 + 2\pi(4)6$

$= 201.06$

$\therefore$ SA is $201.06\ cm^2$

Surface Area of Prisms - Practice

Find the surface area of the following shapes (round to 1d.p. where needed):

a.

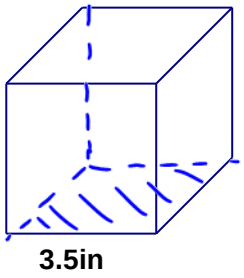
$SA = 6(3.5)^2$

$= 73.5\ in^2$

$V = B \cdot h$

$= (3.5)^2(3.5)$

$= 42.9\ in^3$



b.

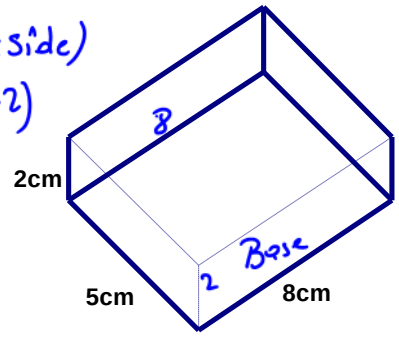
$SA = 2(top + front + side)$

$= 2(8.2 + 8.5 + 5.2)$

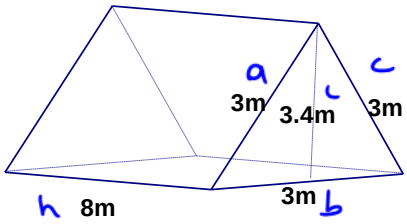
$= 132\ cm^2$

$V = 5(8)(2)$

$= 80\ cm^3$



c.



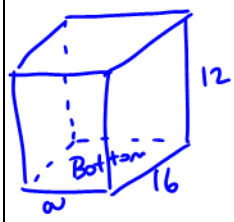
$SA = 3 \cdot 8 + 3 \cdot 8 + 3 \cdot 8 + 3 \cdot (3.4)$

$= 24 + 24 + 24 + 10.2$

$= 82.2 m^2$

$V = B \cdot h = \frac{3(3.4)}{2} \cdot 8 = 40.8\ m^3$

d. A rectangular prism has a length of 16cm and a height of 12m. If the surface area of this prism is $664m^2$, determine the width. and volume



$SA = 2(top + side + front)$

$664 = 2(16w + 16 \cdot 12 + 12 \cdot w)$

$664 = 2(28w + 192)$

$664 - 384 = 56w + 384 - 384$

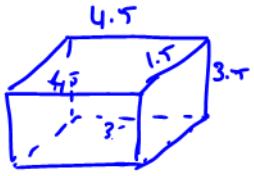
$\frac{280}{56} = \frac{56w}{56}$

$w = 5$

$V = 16 \cdot 5 \cdot 12$

$= 960\ m^3$

e.A toy chest is in the shape of a rectangular prism. Determine the surface area of the toy chest.



$SA = 2(bottom + front + side)$

$= 2(4.5 \times 1.5 + 4.5 \times 3.5 + 3.5 \cdot 1.5)$

$= 2(6.75 + 15.75 + 5.25)$

$= 2(27.75)$

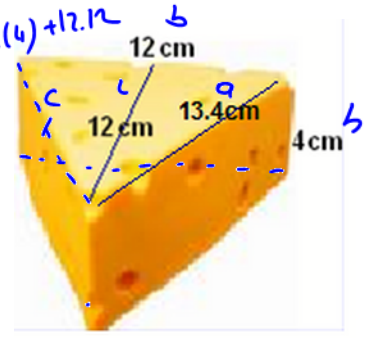
$= 55.5$

f. A piece of cheese is in the shape of an isoceles triangular prism. The cheese needs to be wrapped with saran wrap. Determine the surface area of the cheese to find out how much wrap would cover this cheese.

$SA = (13.4)(4) + (13.4)(4) + \frac{12(4) + 12 \cdot 12}{2}$

$= 53.6 + 53.4 + 48 + 144$

$= 299.2\ cm^2$

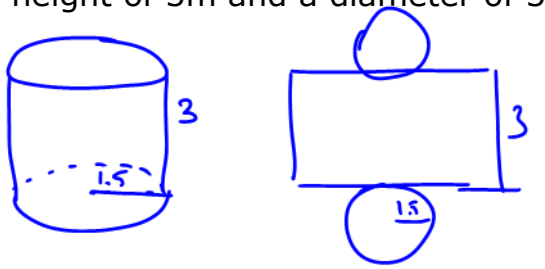


ANSWERS: a. $73.5in^2$, b. $132cm^2$, c. $82.2m^2$, d. 5m, e. $27.8ft^2$, f. $299.2cm^2$

Surface Area of Cylinders – Practice

Find the surface area of the following shapes. Round answers to 1d.p.where necessary. Use 3.14 or the pi button for π.

a.Find the surface area of a cylinder with a height of 3m and a diameter of 3m



SA = 2 circles + rectangle

$= 2\pi(1.5)^2 + 2\pi(1.5)(3)$

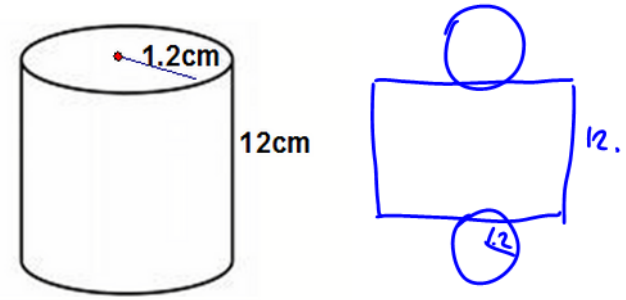
$= 42.4 m^2$

$V = \pi r^2 h$

$= \pi(1.5)^2(3)$

$= 21.2 m^3$

b.



SA = 2 circles + rectangle

$= 2\pi(1.2)^2 + 2\pi(1.2)12$

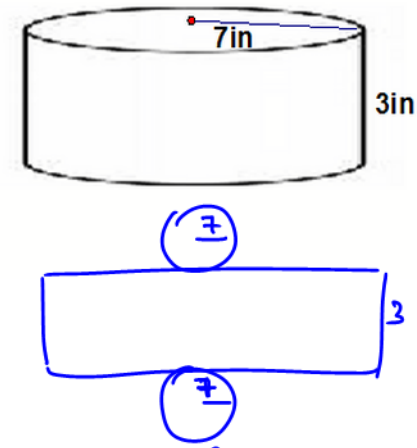
$= 99.5 cm^2$

$V = \pi r^2 h$

$= \pi(1.2)^2 12$

$= 543 cm^3$

c.



$SA = 2\pi(7)^2 + 2\pi(7)(3)$

$= 439.8 in^2$

$V = \pi(7)^2 3 = 461.8 in^3$

d.If a cylinder has a surface area of $178.98 cm^2$, and a radius of 3cm, determine the height of the cylinder.

$SA = 2(\pi r^2) + 2\pi rh$

$178.98 = 2(\pi 3^2) + 2\pi(3)h$

$178.98 = 56.55 + 18.85h$ (−56.55)

$\frac{122.43}{18.85} = \frac{18.85h}{18.85}$

$h = 6.5$

$V = \pi r^2 h$

$= \pi(3)^2 6.5$

$= 183.8 cm^3$

ANSWERS: a. $42.4 m^2$, b.$99.5 cm^2$, c. $439.6 in^2$, d. h=6.5cm