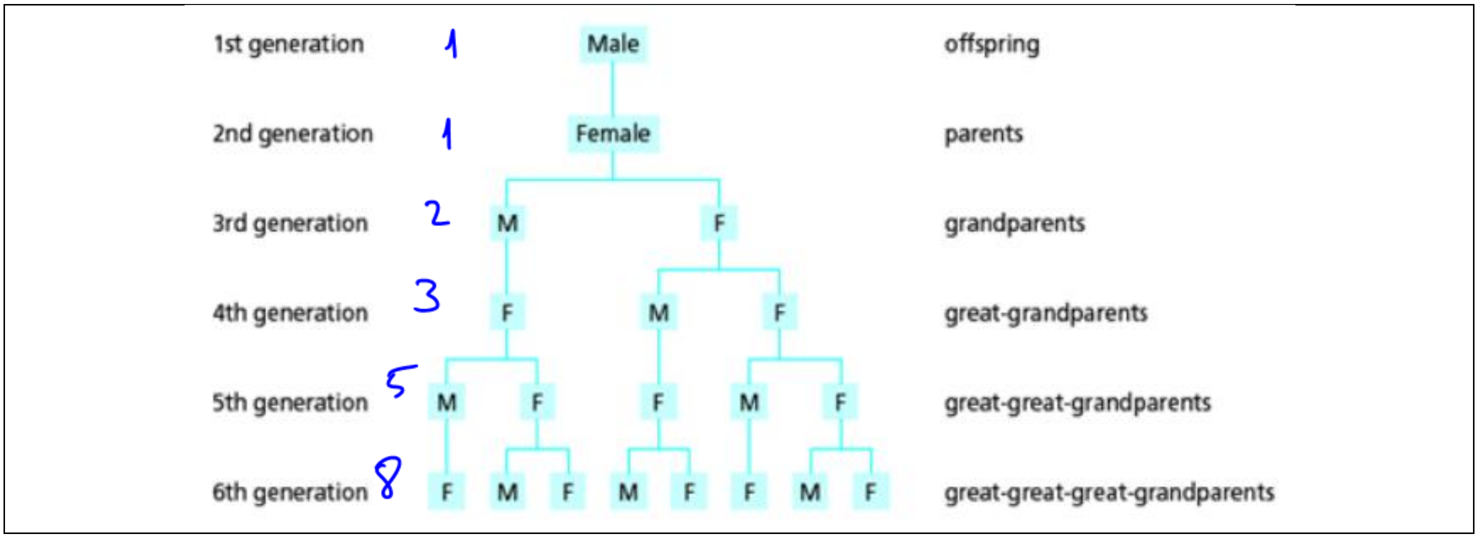
The FIBONACCI SEQUENCE and RECURSION

There are many examples of sequences in nature, and in our everyday lives. A female honeybee hatches from an egg laid by a female honeybee, after the egg has been fertilized by the male. So each female honeybee has two parents. However, a male honeybee hatches from an unfertilized egg and has only one parent, a female honeybee. The tree diagram shows six generations of a typical male honeybee.



1. Starting with the first generation, write the sequence of the number of honeybees in each generation.

1, 1, 2, 3, 5, 8

2. Identify the pattern in this sequence.

To calculate the subsequent terms, we add the previous 2 terms,

3. Use this pattern to determine the next five terms of the sequence.

$t_7 = 5+8 = 13$ $t_8 = 8+13 = 21$ $t_9 = 13+21 = 34$ $t_{10} = 21+34 = 55$

$t_{11} = 34+55 = 89$

4. Let t_n represent any term in this sequence. How could you represent the term before t_n? How could you represent the term before the term that is before t_n?

1, 1, 2

t_{n-2} t_{n-1} t_n

$t_n = t_{n-1} + t_{n-2}$

$t_n - t_{n-2} = t_{n-1} \Rightarrow \boxed{t_{n-1} = t_n - t_{n-2}}$ $\boxed{t_{n-2} = t_n - t_{n-1}}$

5. Given that $t_1 = 1$, $t_2 = 1$, and *n* is a natural number, determine a formula for the general term of this sequence.

$t_1 = 1$ $t_2 = 1$ $t_n = t_{n-1} + t_{n-2}$ $n \in N$ $n > 2$

Key Ideas

- In a **recursive sequence,** a new term is generated from the previous term or terms.
  For example, 1, 2, -1, 3, -4, 7, -11, … is a recursive sequence because each term beginning with the third term, is the result of subtracting t_{n-1} from t_{n-2}.

- A **recursive formula** shows how to find each term from the previous term or terms.
  For example,
  $t_1 = 1$, $t_2 = 2$, $t_n = t_{n-2} - t_{n-1}$, where n is a natural number.
  To find t_n, subtract t_{n-1} from t_{n-2}. The first two terms are given.
  The sequence is 1, 2, -1, 3, -4, 7, -11, … .

- A recursive formula refers to at least one known term. The first term, and sometimes several other terms, appear with the formula. Examine the formula carefully before applying it.

Ex1. Write the first four terms of each sequence.

a) $t_1 = 2$, $t_n = 3t_{n-1} + 5$ $n>1$

$t_2 = 3t_1 + 5$

$t_2 = 3(2) + 5$

$\boxed{t_2 = 11}$

$t_3 = 3(11) + 5$

$\boxed{t_3 = 38}$

$t_4 = 3(38) + 5$

$\boxed{t_4 = 119}$

b) $t_1 = -1$, $t_2 = 1$, $t_n = 2t_{n-2} + 4t_{n-1}$ $n>2$

$t_3 = 2t_1 + 4t_2$

$= 2(-1) + 4(1)$

$\boxed{t_3 = 2}$

$t_4 = 2t_2 + 4t_3$

$= 2(1) + 4(2)$

$\boxed{t_4 = 10}$

$t_5 = 2t_3 + 4t_4$

$= 2(2) + 4(10)$

$\boxed{t_5 = 44}$

Ex2. Write a recursive formula for each sequence.

a) 1, 2, 4, 7, 11, 16 (differences: 1, 2, 3, 4, 5) $t_1 = 1$ $t_2 = 2$

$t_n = t_{n-1} + n - 1$

Check

$t_4 = t_3 + 3$

$7 = 4 + 3$ ✓

$7 = 7$

$t_6 = t_5 + 5$

$= 11 + 5$

$= 16$ ✓

b) 4, 5, 20, 100, 2000, …

$t_n = t_{n-2} \times t_{n-1}$

Check

$t_3 = t_2 \times t_1$

$= 4 \times 5$

$= 20$

$t_5 = t_4 \times t_3$

$= 100 \times 20$

$= 2000$

c) -400, 200, -100, 50, … (ratios: -2, -2)

$t_n = -\frac{t_{n-1}}{2}$

Check

$t_4 = -\frac{t_3}{2} = \frac{-(-100)}{2} = 50$ ✓