Steps	Example #1 $y = -2x^2 - 4x + 3$	Example #2 $y = -5x^2 + 20x + 1$
Common factor the coefficient of the x^2 term from the first two terms. **Do not** factor out the x.	$= -2(x^2 + 2x) + 3$	$y = -5(x^2 - 4x) + 1$
Divide the coefficient of x by 2, and then **square** it.	$2 \div 2 = 1$ * $(1)^2 = 1$ **	$-4 \div 2 = {}^{*}-2$ $(-2)^2 = {}^{**}4$
Add and **subtract** that value inside the bracket of the equation two steps above.	$= -2(\underbrace{x^2 + 2x + 1}_{\text{Perfect Square Tr}} - 1) + 3$	$= -5(x^2 - 4x + 4 - 4) + 1$
Move the last term in the bracket to the outside of the bracket and **multiply** it with the number in front of the bracket. Add the two constants together.	$= -2(\underbrace{x^2 + 2x + 1}_{\text{factored}}) + 2 + 3$ $-2 \cdot -1$	$= -5(x^2 - 4x + 4) + 20 + 1$
Factor the perfect square trinomial inside the bracket.	$= -2(x + 1)^2 + 5$	$= -5(x - 2)^2 + 21$

Vertex $(-1, 5)$ Vertex $(2, 21)$

Practice:

1. Convert the following quadratic relations into vertex form:

a) $y = -3x^2 - 12x + 7$

$= -3(x^2 + 4x) + 7$ $4 \div 2 = 2$, $2^2 = 4$

$= -3(x^2 + 4x + 4 - 4) + 7$

$= -3(x^2 + 4x + 4) + 12 + 7$

$= -3(x + 2)^2 + 19$

Vertex $(-2, 19)$

b) $y = 2x^2 + 10x$

$= 2(x^2 + 5x)$ $5 \div 2 = 2.5$, $(2.5)^2 = 6.25$

$= 2(x^2 + 5x + 6.25 - 6.25)$

$= 2(x^2 + 5x + 6.25) - 12.5$

$= 2(x + 2.5)^2 - 12.5$

$V(-2.5, -12.5)$

2. Determine the coordinates of the vertex of each parabola.

a) $y = x^2 + 8x + 23$

$= (x^2 + 8x) + 23$

$= (x^2 + 8x + 16 - 16) + 23$

$= (x^2 + 8x + 16) - 16 + 23$

$= (x + 4)^2 + 7$

$V(-4, 7)$

b) $y = x^2 - 16x + 44$

$= (x^2 - 16x) + 44$

$= (x^2 - 16x + 64 - 64) + 44$

$= (x^2 - 16x + 64) - 64 + 44$

$= (x - 8)^2 - 20$

$V(8, -20)$

c) $y = -5x^2 + 60x - 187$

$= -5(x^2 - 12x) - 187$

$= -5(x^2 - 12x + 36 - 36) - 187$

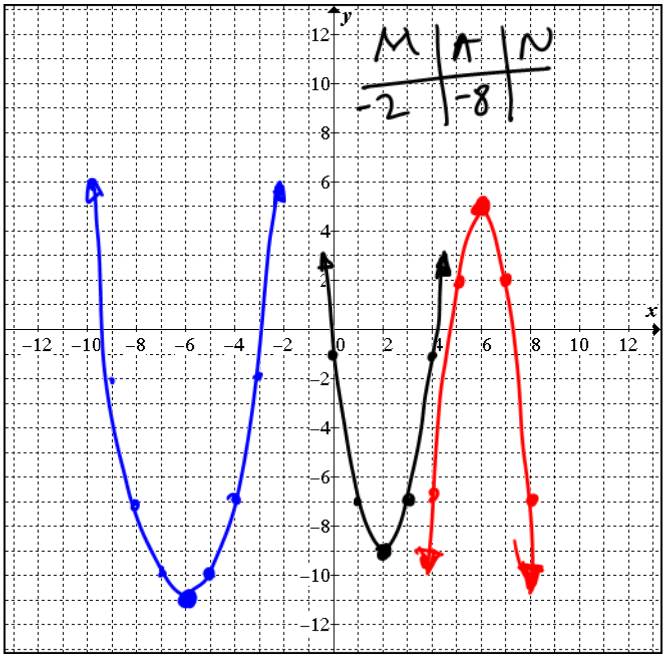
$= -5(x^2 - 12x + 36) + 180 - 187$

$= -5(x - 6)^2 - 7$

$V(6, -7)$

Steps = a × 1, 3, 5

3. Graph each parabola by determining:
   i) its direction of opening and the y-intercept (from the standard form)
   ii) the coordinates of the vertex (by completing the square to obtain the vertex form)
   iii) the x-intercepts (factor or use the quadratic formula to solve the equation $0 = ax^2 + bx + c$).



a) $y = 1x^2 + 12x + 25$ → y-int a=1

$= (x^2 + 12x) + 25$

$= (x^2 + 12x + 36 - 36) + 25$

$= (x+6)^2 - 11$ Vert(−6, −11)

x y

b) $y = 2x^2 - 8x - 1$ → y.int

Steps = a × (1, 3, 5) = 2 × (1, 3, 5)

$= 2(x^2 - 4x) - 1$

$= 2(x^2 - 4x + 4 - 4) - 1$

$= 2(x^2 - 4x + 4) - 8 - 1$

$= 2(x-2)^2 - 9$ Vertex(2, −9)

c) $y = -3x^2 + 36x - 103$

steps −3, −9, −15

$= -3(x^2 - 12x) - 103$

$= -3(x^2 - 12x + 36 - 36) - 103$

$= -3(x^2 - 12x + 36) + 108 - 103$

$= -3(x-6)^2 + 5$ V(6, 5)

4. A ball is kicked into the air. It follows a path given by $h(t) = -4.9t^2 + 8t + 0.4$, where t is the time, in seconds, and $h(t)$ is the height, in metres.

   a) Determine the maximum height of the ball to the nearest tenth of a metre.
   b) When does the ball reach its maximum height?

a) $h(t) = -4.9(t^2 - 1.6t) + 0.4$ $-1.6 \div 2 = -0.8$

$= -4.9(t^2 - 1.6t + 0.64 - 0.64) + 0.4$ $(-0.8)^2 = 0.64$

$= -4.9(t^2 - 1.6t + 0.64) + 2.94 + 0.4$

$= -4.9(t - 0.8)^2 + 2.98$

b) Vertex (0.8, 2.98)

The ball reaches its max height of 3m in 0.8 seconds.